Multiple Games Analysis: A Petri Dish for Growing Polycentric Orders

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Abstract

Game theory, as originally envisioned, is a tool to theorize about social interaction. As game theory became geared towards prediction and control, branching into mechanism and market design theory, its mathematical sophistication focused on solution spaces and, ultimately, the attainment of steady states or simple patterned behavior. I contend that game theory can also describe complex behavior like spontaneous emergence, the development process of polycentric influence hierarchies, and the emergence and survival of institutional forms. Multiple games analysis expands the solution spaces of classic games like the prisoner's dilemma and the principal agent game, not by including conditioning stages or evolving the game in a repeating fashion, but through a detailed expansion of the game itself.

Keywords: game theory; synecology; complexity; agent-based modeling; macroeconomic theory

JEL Codes: E02, C73, D85, C63

> If the whole system of human affairs were subject to systems of polycentric orderings, it would be as though all patterns of order in society were conceptualized as a series of simultaneous and sequential games…
>
> …We might further anticipate that general systems of polycentric orderings applicable to whole systems of affairs would take on the characteristics of competitive games: contestability, innovative search for advantage, and convergence toward successful strategies. If the whole system of human affairs were organized in this way, we would expect to see the emergence of a civilization with greater evolutionary potential than can be achieved by those who call for revolutionary change. (Ostrom & Ostrom 2014: 48)

1. Introduction

Traditional game theory is a framework for strategic interaction where players solve for some strategy or mix of strategies that optimizes utility, profit, payoffs, or some other indicator of well-being. Traditional game theory tends to focus on one-off or multistage and repeated games where all players play all the games under consideration, and where strategies and payoffs within each stage of play are fixed and independent of one another. Traditional game theorists retain this focus to ensure the solution space they're studying is analytically tractable.

Multiple games analysis, to contrast with traditional multistage game theory, studies games whose strategies or payoff functions are somehow interdependent, and where not all players play every interdependent game in the system. Players in a system populated with multiple games do not *a priori* possess the knowledge required to discover their respective optimal solutions at the beginning of gameplay. Furthermore, the system as it evolves may be non-stationary in general, especially if its evolution is open-ended. Even though players cannot generally solve for optimal play conditional on all system-wide relevancies, gameplay is locally constructive, allowing players to act 'rationally' with a restricted information set and to adjust their behavior if it does not properly take into account signals propagated from more distant parts of the system through their local interactions. What players understand about the greater system depends on (a) how games are coupled together, (b) the shape of the social network of gameplay, and (c) how players process and learn from that information as the system evolves, including as new games enter the system and the social network changes its shape.

The theoretical benefit to conducting multiple games analysis comes directly from coupling strategies and payoffs between games and in weakening the heroic knowledge assumptions of traditional game theory, as I will demonstrate below. Coupling games and forcing the social network to be nonintegral can unlock or induce a new set of solutions not available to players in fully-connected systems of independent games. Some anti-social gameplay can be understood in a multiple games context as a *result* of how the system is configured: which games are in the system, which players play which games and when, and how the games are coupled together. As Elinor Ostrom noted, the true prisoner's dilemma requires more players than just the prisoners. One needs a warden to order the solitary isolation of the prisoners; one needs a district attorney to incentivize a warden to obtain confessions in the first place and, arguably, one needs the trappings of social system with the political and financial means to imprison rulebreakers in the first place. With each new player comes new interactions, not between all players in the system, but in local clusters, with payoffs reflective of the concerns of the players at that juncture and likely divergent from the concerns of players at distant junctures of gameplay.

In the current literature, multiple games analysis is typically conducted to investigate the formation and influence of institutions like norms and culture by looking at how mental models used to select strategies in one game can influence how strategies are selected in another (Bednar 2018; Bednar & Page 2018; Bednar & Page 2007). The innovation in multiple games analysis described in this paper and in Devereaux and Wagner (2018, 2019) comes through considering how games are connected to each other by analogizing games as *action arenas* (2008; 2010). I characterize these action areas such that: (i) the outcome of a particular game furnished inputs to another particular game; that is, games are differentially *compatible* and *synergistically coupled* with each other either randomly or specifically; (ii) an agent is required to play a certain sequence of games before attaining some pre-specified goal, that is, the realization of a goal requires the execution of a *plan* characterized by a sequence of synergistically coupled games and a variety of nonintegrally connected players. Many interacting game sequences with particular interdependencies form an *ecology of games* (Long 1958; Wagner 2012; Lubell 2013; Smaldino and Lubell 2014). Define a *synecological game* as a particular chain of synergistically coupled games such that no player plays all the games in the chain. An ecology of synecological games is a *synecological system*, defined in more detail below.

I contend that synecological systems theory as a theoretical framework allows the theorist to do far more than contextualize a prisoner's dilemma. This form of multiple games analysis is a candidate theory for microfounding macroeconomic theory, proper. The advantages of synecological systems theory over current ways to microfound macro theory are: (1) it is capable of describing emergent, spontaneous and polycentric orders (Devereaux and Wagner 2018, 2019) (2) it allows for the endogenization of contextual choice without the need for the theorist to analytically derive a solution space, meaning that contextualizing a prisoner's dilemma is as simple as adding new games and players and relationships in, say, an agent-based simulation where such additions are simple; (3) it blurs the line between "public" and "private" activities and interactions, encouraging theorists to include all relevant interactions in a model of a particular sphere of interest instead of leaving out how public intervention in particular effect private behavior; (4) its knowledge assumptions are weak; (5) it is designed to be mathematically constructive at the system level and to require the use of computational simulations to study system-level characteristics for any reasonably sized system, thus avoiding the computability problems that increasingly plague traditionally microfounded macro theory (Lewis 1985; Velupillai 2007).

A crisis of economic theory arose in the wake of the global financial recession, which state-of-the-art macroeconomic theory had both failed to predict and, afterwards, to explain (Goodfriend 2007; Mishkin 2007; Krugman 2011; Romer 2016; Stiglitz 2018). But 2007-8 wasn't the beginning of this crisis in economic theory, it was merely a punctuated and embarrassing example of the ongoing crisis. The seeds of the crisis lay in the Sonneschein-Mantel-Debreu (SMD) proof that the functional forms of aggregate demand functions were far less restricted than required for general tractability (Sonneschein 1973; Mantel 1974; Debreu 1974). Therefore, as was proved in the decades after the result, aggregate demand functions need not have unique equilibria, be stable, or allow for comparative statics and econometric identification (Rizvi 2006). SMD called into question the validity of a microfounded macro theory that uses aggregation to force a direct relationship between analytically unique equilibria at the individual level and analytically unique equilibria at the macroeconomic level. If aggregate demand functions need not have analytically unique equilibria then a theory of individual choice that restricts choice behavior to those choices which produce analytically unique equilibria seems over-restrictive at best, and at worse, plain wrong.

Where precisely the problem enters theoretically has been the central issue these four-and-a-half decades since the SMD result, though as Rivzi (2006: 230) notes, the hunt for new theories didn't really begin until the 1980s. Below is a point-by-point summary of the theoretical insights about a plausibly microfounded macro theory.

People Choose in Contexts and Form Relationships: Economic theory ignores the context in which economic decisions are taken at its own peril. Ignoring choice context misses the out-of-model ways in which people interact to exploit gains from trade and sets up models to fail in unpredictable and possibly spectacular ways. (Farmer and Foley 2009; Kirman 2014; Haldane & May 2011; Helbing & Kirman 2013).

Mathematical Economics Needs to be Able to Describe Processes: Economic theory at its own peril glosses within-plan choice processes by insisting on a mathematics suitable for reversible thermodynamic systems, what Stuart Kauffman calls the Newtonian paradigm (Kauffman 1999). The implications of departing from the Newtonian paradigm include, perhaps most importantly: no more "as if" theorizing (as in Friedman 1953), and an explicit recognition of the nonstationary "diachronic" nature of complex evolving social systems (Shackle 1972, 1974; Potts 2000; Farmer & Foley 2009; Kirman 2010; Axtell 2005)

Knowledge is Never Complete: To have a plausibly endogenous theory of growth and cycles, one must acknowledge the asymmetry of time and knowledge (Hayek 1945; O'Driscoll, Rizzo & Garrison 1996; Bergson 1908; Shackle 1972; Koppl et al 2015). The states of socioeconomic systems are fundamentally unknowable at longer timescales, exhibiting what Longo et al (2012) and Zia et al (2014) characterize as "unpredictability and unprestatability."

Macroeconomic Patterns May Be Irreducible: Socioeconomic systems exhibit fundamental complexity that makes a great deal of what's going on in the system irreducible to anything resembling the aggregation of the characteristics, actions, and goals of representative agents (Farmer & Foley 2009; Beckage et al 2013; Miller and Page 2009; Kirman 2010; Arthur 2013; Axtell 2005).

Rationality is Heterogeneous, Heuristic, and a Property of the System: Choosing agents are embedded in environments that can affect how they act and what they want. Knowledge and the effectiveness with which agents associate actions with some set of observational percepts is bounded and typically heuristic (Simon 1996; Gigerenzer & Todd 1999). Combined with the other points above, rationality as it coheres to form patterns at the macroeconomic level is a property of the system and not merely the aggregation of agent choice. (Smith 2003; Wagner 2012; Farmer & Foley 2009; Kirman 2010)

Can game theory be a social theory that embraces the points above? Yes, though not perhaps in its current overstrong and analytically-closed form. In the pre-Nash era, game theory focused more on coordination games, coalitions, and what was called the core of game theory, namely, Pareto optimal solutions to gameplay. As game theory became geared towards prediction and control in the wake of the Nash proof (1950), it eventually morphed into the fields of mechanism design (Hurwicz 1973) and market design (Roth & Wilson 2019). Theorists took up the question of the computability of equilibria, particularly in n-person games with multiple equilibria, and proved in general that it is computationally "hard" to construct the equilibria in these games (Papadimitriou 1994; Daskalakis et al 2009; Babichenko and Rubinstein 2016).

Discovering these equilibria is neither algorithmically feasible for individuals or expert like oracles; it is, therefore, unreasonable to base a decision theory on their discovery. This places traditional game theory in a bit of a bind. What happens to game theory if we reject the necessity of both the predictability and agent-solvability of games?

Games remain a way of describing the entanglement between agent actions and action-outcomes, and the actions and action-outcomes of others, at least *locally*. Furthermore, we can describe the way in which system information reaches individual players under conditions of strictly local knowledge if outcomes and actions are allowed to change through time in a way that reflects the contextual realities of distant games despite weakening integrability and complete knowledge. In this ecology of games, players need not know all strategic and payoff interdependencies in order to make decisions.

We do, however, lose a few methodological advantages in the process. System behavior is no longer a direct reflection of the process of individual decision-making. This means, for any but the smallest synecological games, that the modeler needs to specify all the games and dependencies in an agent-based evolutionary format in order to get a sense for how and whether the system resolves into patterned behavior after a period of time. The advantage of specifying the context of choice in the form of interdependent games comes with the disadvantage that we cannot cleanly separate "market" and "public" orders in the system. Players instead gain or lose membership in various games, some of which may be publicly ordered, and others privately ordered. Any influence flowing from public to private, or vice-versa, must be specified in terms of game membership and interdependencies between games.

Synecology is a term borrowed from theoretical ecology to mean the study of a group or community of organisms as opposed to *autecology*, which refers to the study of an individual organism. Synecological systems theory is to representative agent theory as the study of a community of organisms is to the study of a sum of individual organisms. Summing up over representative agents leaves unanswered the question of how institutions arise from the interaction of individuals, a question often left to the designer to answer by assumption or by inferring patterns from data. The gold standard of microeconomic theory is to divorce situations from wider contexts ('ceteris paribus') to find laws that hold true regardless of context. Such theoretical architecture is suitable for studying simple situations closed off and independent from the greater realm of economic activity. But it is inappropriate for studying macroeconomic systems. Game theory was built to be a language of interdependencies; multiple games analysis can provide a bridge between traditional game theory and a new kind of social theory.

2. Synecological Game Theory: Definitions and Examples

Synecological systems are systems whose agents may engage in simpler underlying behavior that, when conjoined with the behavior of other agents, produce complex, unprestatable system states. Synecological implies *synergy*, or what Bob Coecke (2017) calls "togetherness." The synergy (togetherness) of combining abstractions foo_1 and foo_2 is, a Coecke put it, "the new stuff that emerges when foo_1 and foo_2 get together" (ibid: 63). Synergy is a key and often unexplained feature of socioeconomic systems.

Suppose an agent *i* plays a game with agent *j*, then plays another game with agent *k* such that features of the first game—its payoffs, or perhaps how it is played—determines something about the features of the second game. Suppose then that *k* plays a game with *l* such that features of this game determine something about the features of the game between *k* and *j*. The coupling of the first and last two games results in different gameplay than if each of those pairs had been independent from one another; more importantly, the way *j* and *k* play games with *i* and *l* creates a link of influence between *i* and *l* even though *i* and *l* do not play directly with one another, and presumably, may not even know of the other's existence.

Synecological systems 1) allow agents to utilize system knowledge indirectly by engaging with their own local knowledge sets, and 2) subject agents to the effects of rules-following by distant agents, with whom they may never interact directly.

I briefly introduce the structure of synecological game theory. I then discuss two examples of synecological games that will serve to illuminate our analysis throughout the rest of the paper.

Definition 1: An **extended game** is an *m*-length chain of games γ_i with *N* heterogeneous players, where not all players necessarily play every subgame in sequence, and such that at least 1 player is shared between sequential subgames in the extended game. Extended games are a generalized version of multistage and repeated games, where stages 1) can be of different dimensions, and 2) need not have more than 1 player in common.

Definition 2: A **synecological game** $\Gamma = \{\gamma_1, \ldots, \gamma_m\}$ is an extended game of length *m* where the following conditions hold:

(i). *Weak Connectivity* - At least one player, but not all players, are shared between sequential games

(ii) *No-dictator Condition* - No single player spans the entire extended game

(iii). *Entanglement* - Suppose the shared player *i* plays subgame γ_{j-1} first, and subgame γ_j subsequently. Then, the payoffs v_i^j of subgame γ_j are a function of the payoffs of the previous game v_i^{j-1} and the strategic actions of the nonshared players in γ_j.

Theorem 1: A *minimal* synecological game Γ_{MIN} has *N* = 4, of which 2 are shared players. The number of subgames in the minimal synecological game is $|\Gamma_{\text{MIN}}| = 3$.

Theorem 2: A minimal strategy space $\mathcal{A}$ of the minimal synecological game Γ_{MIN} is the 64-member tensor product $A^1 \otimes A^2 \otimes A^3$ of the strategy spaces of each game, respectively.

Theorem 3: The strategy space experienced by any shared player *j* is a tensor product of the strategy spaces in the games in which *j* participates. So, if *j* participates in a subset of games $\mathcal{G}_j \in \Gamma$, where Γ is the synecological game, we can associate a set of strategy spaces $\mathcal{S}_j$ with $\mathcal{G}_j$. Then, the strategy space utilized by j can be represented as $\boldsymbol{\mathcal{S}}_j = \{\otimes \mathcal{S}_j^k\}_k$ where k indexes the members of $\boldsymbol{\mathcal{S}}_j$.

Synecological games have graphs that encode the directionality of subgame play within the synecological game. An example minimal synecological game with subgames $\gamma_1, \gamma_2, \gamma_3$ can be written in normal-form as (EQ1):

$$\Gamma = \langle \{\gamma_1 \rightarrow \gamma_2 \leftarrow \gamma_3\}, N = \{\{1,2\},\{2,3\},\{3,4\}\},$$
$$\mathcal{A} = A^1 \otimes A^2 \otimes A^3, \{v_1^1(a_1), v_2^1(a_1), v_2^2(v_2^1(a_1), a_2),\ v_3^2(v_3^3(a_4), a_2), v_3^3(a_4), v_4^3(a_4)\}\rangle$$

whose graph looks like:

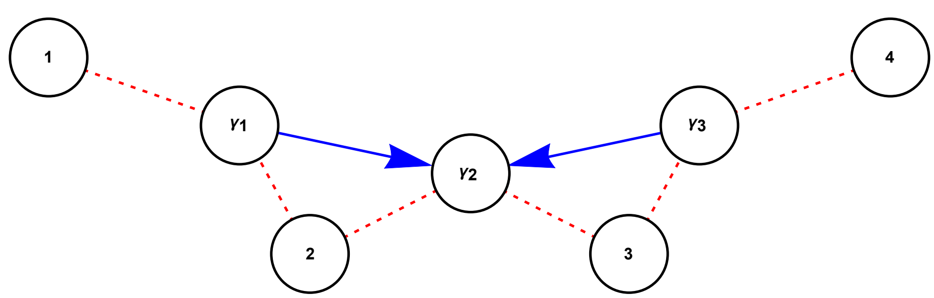

Figure 1: The graph representation of the above synecological game.

and whose coupled payoff functions for an example strategy profile $s = \{((a_2)_1^1, (a_1)_2^1), ((a_2)_3^2, (a_1)_4^2), ((a_2)_1^3, (a_1)_3^3)\}$ of the 64-member strategy space look like:

$$V_1 = \{v_1^1((a_2)_1^1, (a_1)_2^1), v_2^1((a_2)_1^1, (a_1)_2^1)\}$$

$$V_2 = \{v_3^2((a_2)_3^2, (a_1)_4^2), v_4^2((a_2)_3^2, (a_1)_4^2)\}$$

$$V_3 = \{v_3^3(f(v_3^2((a_2)_3^2, (a_1)_4^2)), (a_2)_1^3, (a_1)_3^3), v_1^3(g(v_1^1((a_2)_1^1, (a_1)_2^1)), (a_2)_1^3, (a_1)_3^3\}$$

where the parenthetical a_i refer to the two actions available to each player, the bottom index outside the parentheses indicates which player is acting, and the top index indicates the game being played. Each action pair represents a strategy profile in their respective subgame. The functions *f*, *g* couple the payoffs of subgames for the shared players 2, 3 respectively, and where v_n^m represents the payoff function for player *n* in game *m*.

Let's consider a couple examples of synecological games. The first game we cover shows how a prisoner's dilemma can be understood as a *product* of its contextualization, not the starting point of analysis. Therefore, when finding or suspecting prisoner's dilemmas in the wild, it is (or should be) incumbent on the theorist to describe how the prisoner's dilemma is being supported by its institutional context, or look for solutions in its institutional context as to why it is *not* present if suspected but not found (as in the Ostroms' empirical investigations into self-organized solutions of CPR problems). The second game we cover illustrates how the solution space of a principal agent problem expands when coupling two "production plans" together by means of simple competition, thereby incentivizing agents to exert higher levels of effort even in the absence of perfectly enforceable employment contracts. In short: principal agent problems, when embedded in a market context, cease to become problems.

2.1 A simple prisoner's dilemma

As noted in McAdams (2008), there may be a sociology of science reason for why theorists find prisoners' dilemmas so analytically popular: 1) the prisoner's dilemma has a single equilibrium, quite rare in games in general; 2) the prisoner's dilemma seems to imply fundamental interactional failure and thus provides a strong case for third party intervention.

What I'm interested in is not the analytical implications of existing prisoners' dilemmas, but how prisoners' dilemmas might arise. I start with the classical prisoner's dilemma. The key situational assumption that reinforces the anti-social equilibrium is that prisoners are unable to communicate with each other. But how does this situation come to pass? By explicitly endogenizing the situation at hand, I believe we can learn something about how prisoner's dilemmas come to be, how they are avoided, and how to subvert them if we are presented with one.

In a hypothetical synecological prisoner's dilemma game, *confessions* could be a variable input to, say, the warden's game with the district attorney to obtain more money for the prison in competition with other wardens under the district attorney's jurisdiction.

Let's extend the prisoner's dilemma into a synecological game by considering all the essential players and games required to endogenize the anti-social solution, and the interdependencies between the players and games. First, we add two more players to the original two-player game: the warden and the district attorney. Now, let's consider the games that generate the incentives required for the warden to come upon parallel solitary confinement (SHU) as the best way of generating the anti-social solution we know as the prisoner's dilemma. We start with the district attorney.

Perhaps the district attorney is running for office and is hopeful that more convictions will appeal to their prospective constituents. Suppose the district attorney controls funding to prison system, and promises more funding to prisons which obtain more confessions from prisoners. More funding means more income and prestige for the wardens whose prisons obtain it. Given that the warden is incentivized to produce more confessions and has a large amount of control over the institutional environment in which prisoners find themselves—having control over access to privileges and housing arrangements, for instance—what is the best way for them to obtain more confessions?

In a synecological game, especially an agent-based game with many rounds of play, wardens would not necessarily know ahead of time but could discover that given the choice between keeping prisoners in the general population and putting them in solitary, splitting up collaborators generates a higher percentage of confessions than not. We see once we expand and contextualize the game using multiple games analysis that the prisoner's dilemma *emerges* as a solution due to the specific institutional realities built into the game: (a) the costs of staying mum are relatively low in the general population where behavior prior to interrogation can be coordinated and relatively high in SHU; (b) prisoners have fewer rights and can be subject to solitary confinement; (c) that prison is a dangerous environment, allowing for credible threats of retribution via allies *outside* prison to be made if suspected collaborators remain in the general population before interrogation. In short, prisoners find themselves in an anti-social dilemma because of a lack of agency as prisoners, the existence of SHU, the exigent political and financial incentives to be put into SHU, and an epistemologically myopic view of the game at the system level.

Next, I take a much more detailed look at a synecological game that encodes incentives in the form of two coupled principal agent games. The prisoner's dilemma is also a solution to the following game, but rather than being built into the institutional framework of gameplay, it is an emergent solution that exists by virtue of the synergistic entanglement of these principal agent problems via market competition.

2.2 The "incentive" synecological game

In his 2009 book *Game Theory Evolving,* Herbert Gintis introduces an extended game he calls "The Allied Widgets problem" with two arenas of action, two principal agent (PA) games between a rival company owners (Allied and Axis Widgets, respectively) and their managers who are tasked with discovering the lowest-cost method of producing their final product, and a Cournot competition game between the two owners. Nature is also a player. The PA games happen first, between each respective owner and manager. No information flows between the two PA games at this stage.

The internal structure of the principal agent problem faced by each manager is (Gintis: 83):

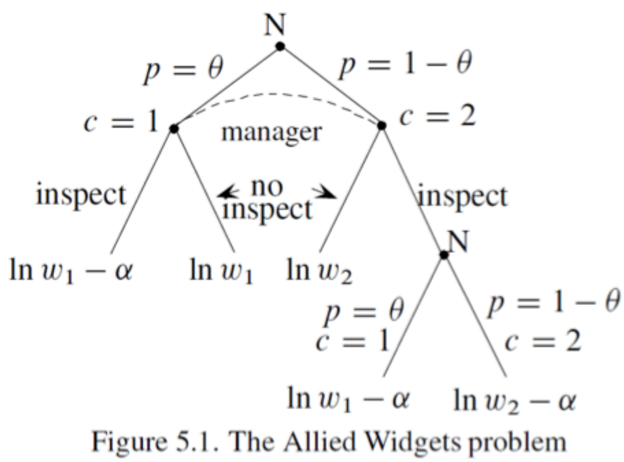

Figure 2: The Allied Widgets problem, managerial decision tree

Nature (N) 'sets' the marginal cost (c) of using one production technique, e.g., fusion vs. fission. The manager employs costly search to inspect the state of Nature ($p = \theta$ is the probability of the current/inspected state to have marginal cost $c = 1$). Inspection cost is represented as α. The owner observes c, but cannot observe whether the manager inspected or not. If $c = 1$, then the manager pays a higher wage w_1, and if $c = 2$ the manager pays the lower wage w_2. The reservation wage is w_0, and, generally, $w_2 > w_1 > w_0$.

Can the owner *choose* values of w_1, w_2 that somehow incentivizes the manager to always inspect? In the isolated principal agent game, there doesn't seem to be a way to do this. But suppose we explicitly market competition between the two owners—who both sell widgets—as a game. Let demand be some exogenous function of total market quantity of widgets, $P(Q) = a\,Q + b = a(q_1 + q_2) + b$, where q_1 is the quantity of widgets produced by the owner of Allied Widgets, and q_2 is the quantity of widgets produced by the owner of Axis Widgets. Each owner *i* then faces a profit maximization problem: $profit_i = revenue_i - costs_i = (a(q_i + q_j) + b)q_i - (c_i + w_i)$, where q_i is the variable of choice in the short run, but where w_i is an evolutionary variable of choice.

The solution space is technically infinite-valued, as w_1, w_2 in each game are effectively real-valued. However, we can determine the solution space by making assumptions about the game. We assume that managers have some kind of reservation wage w_0 under which they will not work, thus creating a wage-incentive floor. We assume both games are symmetric, $(w_0, w_1, w_2)_1 = (w_0, w_1, w_2)_2 = (w^*, w^-, w^+)$. We assume that Nature provides the same environment of marginal cost realities $p = \theta$ for each owner/manager pair, that managers cannot collude, and that managers have identical utility functions and effort α. Given this rarified setup, we can deduce an incentive compatibility constraint that allows owners to choose (w^*, w^-, w^+) in a way that effectively changes the game from one where the managers' fates had been decoupled, into a game where the managers' fates are coupled in a prisoner's dilemma, inducing interaction-at-a-distance (Gintis 2009: 86):

	Inspect	Shirk
Inspect	$\ln w^* - \alpha$ $\ln w^* - \alpha$	$\phi \ln w^* + (1-\phi)\ln w^+ - \alpha$ $\phi \ln w^* + (1-\phi)\ln w^-$
Shirk	$\phi \ln w^* + (1-\phi)\ln w^-$ $\phi \ln w^* + (1-\phi)\ln w^+ - \alpha$	$\ln w^*$ $\ln w^*$

Table X: An induced prisoner's dilemma that incentivizes players to defect ("inspect")

where $\phi = 1 - \theta - \theta^2$ is the probability that both managers choose equal-cost technologies when one manager chooses *inspect* and the other chooses *no inspect*. Weakening any of these assumptions destroys Pareto optimality.

We can write the Allied and Axis Widgets game as a synecological game, which we shall call the incentive synecological game. The graph is below. Solid blue lines indicate directional relationships between games, and dotted red lines indicate agent membership in gameplay.

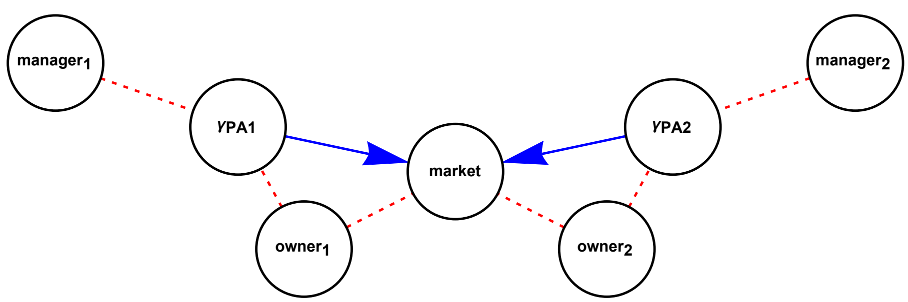

Figure 3: The incentive game as a synecological game

We notice that the abstract structure of the incentive synecological game is basically identical to EQ1:

$$\Gamma = \langle \{\gamma_1 \to \gamma_3 \leftarrow \gamma_2\}, N = \{\{1,2\},\{2,3\},\{3,4\}\}, \mathcal{A} = A^1 \otimes A^2 \otimes A^3,$$
$$\{v_1^1(a_1), v_2^1(a_1), v_2^2(v_2^1(a_1), a_2),\ v_3^2(v_3^3(a_4), a_2), v_3^3(a_4), v_4^3(a_4)\}\rangle$$

where $\gamma_1 = \gamma_{\text{PA1}}, \gamma_2 = \gamma_{\text{PA2}}, \gamma_3 = \text{market}$. 1 = manager$_1$, 2 = owner$_1$, 3 = owner$_2$, 4 = manager$_2$, $(a_1)_2^1 = (a_1)_4^2 = no\_inspect$, and $(a_2)_2^1 = (a_2)_4^2 = inspect$. Recall that $(w_0, w_1, w_2)_1 = (w_0, w_1, w_2)_2 = (w^*, w^-, w^+)$. Then, the expected payoffs to each manager if they don't search versus if they do are:

$$v_{2,4}^1\big((w^-, w^+), \text{no\_inspect}\big) = \theta \ln w^- + (1-\theta)\ln w^+$$

$$v_{2,4}^1((w^-, w^+), \text{inspect}) = [1-(1-\theta)^2]\ln w^- + (1-\theta)^2 \ln w^+ - \alpha$$

The payoff to each owner in each principal agent game is more of an outcome: the cost of the production process used, C. That cost relates to the payoffs in the final game by its inclusion in the profit function $v_i^3 = \pi$. Recall that the cost of production is the marginal cost of production plus the wage paid to the employee.

$$\mathrm{g}\left(v_{1,3}^1\big((w^-, w^+), \text{no\_inspect}\big)\right) = \mathrm{C}\left((w^-, w^+), \text{no}_{\text{inspect}}\right) = \theta(w^- + 1) + (1-\theta)(w^+ + 2)$$

$$\mathrm{g}\left(v_{1,3}^1\big((w^-, w^+), \text{inspect}\big)\right) = C\ ((w^-, w^+), \text{inspect}) = [1-(1-\theta)^2](w^- + 1) + (1-\theta)^2(w^+ + 2)$$

The final payoffs to each owner are revenue minus costs, which are functions of wages, marginal cost, and the quantities sold by each owner. So, for owner *i* playing with owner *j*:

$$\begin{aligned} v_i^3\big(\mathrm{C}\big((w^-, w^+), \text{no\_inspect}\big), q_i, q_j\big) \\ &= \pi_i\left(\mathrm{C}\left((w^-, w^+), \text{no}_{\text{inspect}}\right), q_i, q_j\right) \\ &= \left(a\big(q_i + q_j\big) + b\right)q_i - [\theta(w^- + 1) + (1-\theta)(w^+ + 2)] \end{aligned}$$

$$v_i^3\big(\mathrm{C}\big((w^-,w^+),\text{inspect}\big),q_i,q_j\big)=\pi\big(C\big((w^-,w^+),\text{inspect}\big),q_i,q_j\big)$$
$$=\big(a\big(q_i+q_j\big)+b\big)q_i-([1-(1-\theta)^2](w^-+1)+(1-\theta)^2(w^++2))$$

In an evolutionary synecological game, agents progress through many rounds of play and use heuristic ranges to test how profits and utilities respond to different values of the variables over which they have influence. In the incentive synecological game, owners choose the wage differentials. So even if it is rather unlikely for owners to solve for wages that induce a prisoner's dilemma given the abstract expected payoffs above—particularly as in even a somewhat realistic system owners cannot expect the system to be idealized and symmetric—owners will have many rounds to discover the wages that induce a prisoner's dilemma between their respective managers.

3. Synecological Game Theory: Abstracting the Theory

3.1 Endogenous inputs and outputs, and coupling functions

Synecological game theory is characterized functions that couple outcomes of two games. In order to couple the outcomes of two games, variable(s) are passed from a previous game to be used in the coupling function of the subsequent game. For the incentive synecological game, wages (w) and marginal cost (c), both determined in the first two principal agent games, are passed along to be used in the determination of profits in the subsequent market competition game. Call these kinds of endogenous variables *inputs* when they are imported from a previous game to be used in the determination of the payoffs of the current game, and *outputs* when they are exported from the current game to be used in the coupling function of a subsequent game.

Coupling functions, therefore, have domains that are at least as large as the dimension of the set of inputs into the current game. In the market competition Cournot game of the incentive synecological game, the domain of the coupling function—the payoff function that was symmetric between both owners—had four-dimensional domain $R_4 = Q \times Q \times \mathrm{MC} \times W$, where Q is the set of all feasible quantities that each widget company can produce, MC = {1, 2}, and W is the set of all feasible wage values. The range of the coupling function in the Cournot game was the real numbers.

We begin to see, then, that coupling functions represent real technologies, synergies, and other kinds of relationships between variables whose values are determined during the process of plan-realization. Suppose we add a new intermediate stage to the production process exemplified in the incentive synecological game, say, an interaction with a public official to place constraints on competitors or acquire subsidies that then alter how the production process is transformed into profits. The domain of the coupling function that determines profits in the final stage of the game may then expand to include the subsidies, and the Cournot game might then *output* a variable that is then inputted into a new, political game, perhaps a percentage of profits or some other kickback. Thus we see how what seems like a market competition game can transform into a hub whose wheel-spokes lead to a variety of opportunists, who exist by virtue of the institutional environment.

3.2 Coupling functions and traditional game theory's "identity" coupling function

The simplest coupling function is $f(\vec{a}) = \vec{a}$. We call this coupling function the identity coupling (IC) function. This is the coupling function employed in standard game theory, where games, even subgames in multistage games, have internally independent payoff functions. In multistage games, we can write a big coupling function that is the sum of all the intermediate coupling functions. The following is the overall payoff for agent *i* in a traditional multistage game with *M* stages, where *f* is the IC coupling function over all the stage-based payoffs, v_i^j is the payoff function for agent *i* at stage *j*, and s_j is the strategy profile at stage *j*:

$$V_i = f_i(s_1, s_2, \dots, s_M) = f_i(\sum_{j=1}^{M} v_i^j(s_j)) = \sum_{j=1}^{M} f_i^j(v_i^j(s_j)) = \sum_{j=1}^{M} v_i^j(s_j)$$

As expected, the overall payoff in a multistage game is the sum of the payoffs at each individual stage. This is *not* the case for synecological game theory. Consider the incentive synecological game. The coupling function in this case is a function of how the input variables affect revenue minus a function of how the input variables affect costs. That is, $f_{\mathrm{PC}}(\vec{a}) = R(\vec{a}) - C(\vec{a})$, where "PC" means "profit coupling." Define $\vec{a}$ as the vector of all variable values at then end of synecological gameplay. For the incentive synecological game, this is: $\vec{a}_{\mathrm{Inc}} = \{w, \mathrm{MC}, q_1, q_2\}$, where the values of *w*, MC are determined in γ_1, and the values of q_1, q_2 are determined in γ_3. Then, in our example,

$$f_3\left(\vec{a}\right) = f_3(w, \mathrm{MC}, q_1, q_2) = R_3(w, \mathrm{MC}, q_1, q_2) - C_3(w, \mathrm{MC}, q_1, q_2)$$
$$= q_1(a\,(q_1 + q_2) + b) - (q_1 * \mathrm{MC} + w)$$

There are many other conceivable coupling functions. Consider a matching game that couples the 'production processes' of the plans of *N* people. Examples could be marriage/dating markets, or academic job markets. The coupling function in this case utilizes a variety of variables outputted during the plan realization process, as well as direct strategies employed during the matching game itself.

We can also consider coupling functions that represent measurable but non-monetized goals, like in Schelling segregation, where people act to eventually realize a situation whereby the fraction of people like them is greater than or equal to some $l \in [0,1]$. For instance, we could consider a coupling function that demonstrates a kind of Schelling-esque discrimination in employment. A synecological segregation game might, for instance, look at segregation within and between fields. In the first game, students act in departmental clusters to form hierarchies within the clusters. In the second game, students play with departments who are hiring, and who rate along two metrics: merit, and similarity (intellectually and demographically) to the median characteristics of the department. The coupling functions in this case draw on some characteristics determined during the first stage of play to form the Schelling similarity metric. An evolutionary agent-based synecological model might allow for students to form expectations of discrimination depending on the field or department, and we could model intellectual and demographic sorting within and between fields.

3.3 Topological considerations: the compatibility of games, social networks, and influence

In order to evaluate the success or failure of plans at any step, plans need to resolve. Topologically, this means we can have no cycles in our game relationship graphs. Note in particular that the shared players in the minimal synecological game are particularly influential players, the bridges between pairs of games. Shared players have the opportunity to leverage gameplay in other action arenas to channel the synergistic gains from coupled games to themselves. Therefore, a systems-level model of multiple games, like an agent-based evolutionary synecological game, should include a way of talking about the network macrostructures (like clusters), microstructures (like in-stars and transitive triads), and include measures of node-level influence both when looking at the social network and the game-level network.

As Bednar (2018: 3) notes, multiple game analyses require the modeler to specify how games are related in the agent's mind, or in reality. In Bednar (2018) and Bednar and Page (2016, 2007), games are related by way of conditioning behavior. That is, games themselves can occupy unrelated arenas of action, whose only link is that the same agent plays them. In synecological game theory, games are explicitly related, with the outputs of some games being inputs to other games. Certain games are compatible with other games. Define a compatibility set C_{γ}^{f} in the space of all games relative to any particular game γ and coupling function *f* in the system. These are games $c \in C_{\gamma}^{f}$ who output the right number and type of variables needed to satisfying the necessary input variables of γ given some coupling function *f*, or vice-versa.

We see immediately that the coupling of plans must happen in a way that respects the compatibility between the coupled games given some coupling function. This makes synecological systems theory somewhat resemble the theory of generalized autocatalytic sets (GACS) in biochemistry. The modeling of production processes as autocatalytic sets is a current research area of complexity economics (Padgett 1996; Steel 2000; Padgett & Collier 2003; Padgett et al 2012; Zia et al 2014). In an unfinished working paper, I explore the mathematical relationship between GACS and a system of synecological games. So far, there are great similarities between the two. Synecological game theory could be a way of bridging between traditional microeconomic theory and the microfoundations of a new kind of macro systems theory.

3.4 Are synecological games reducible to multistage games?

We should be clear about the differences between multistage games and synecological games. The definition of a multistage game requires that agents are able to observe the history of action after each stage, even if they are technically doing nothing during that stage (Fudenberg & Tirole: 70-1). In contrast, synecological games *hide* the history of some stages from other agents. In a minimal synecological game, agents may be able to engage in some kind of pattern classification on observed data to determine how it is affected by gameplay outside their direct observation, and may get close enough to infer some aspects of relevant gameplay outside their plan when they are strategizing in-plan.

Let's write it in the Fudenberg-Tirole terminology. Let the set of all agents be $\mathcal{I}$, the action profile at stage *k* to be a^k, and the history at the end of stage *k* to be $h^{k+1} = (a^0, a^1, \dots, a^k)$. The set of actions available to player *i* at stage *k* generally depends on what happened before stage *k*, so then $A_i(h^k)$ denotes the possible *k*-stage actions available to player *i*. Now, synecological games *do* have histories. Suppose we have a three-member synecological game where $\gamma_2 \rightarrow \gamma_2 \rightarrow \gamma_3$. Then the history once game 3 is played will be $h^3 = (a^0, a^1, a^2)$. However, player 1 only sees (a^0), player 2 sees (a^0, a^1), player 3 sees (a^1, a^2) and player 4 sees (a^2).

These limited histories do not allow players to solve for optimal solutions as if they had the entire history, as they are not aware of all the dependencies, and the system may very well be irreducible to a simpler representation of itself. We see an example of a system irreducible to a more standard characterization proved quite elegantly when games are played on networks, as in Jackson & Zenou (2015:116-122), where the authors show for a game of positive externalities in effort expended that agents will generally undersupply the effort needed to attain a social welfare optimum if gameplay had not been on a less-than-complete network.

4. Evolutionary Synecological Game Theory is Agent-based

Synecological game theory bakes in epistemological limitations at the agent-level. Agents need to learn from patterns in the real-valued metrics of the success or failure of their plans over many iterations if they hope to choose values for variables they control that make them the best off they can be. Furthermore, there is no reason in agent-based synecological game theory that games themselves can't be objects of choice and discovery. Agents realize plan-ends; the synecological game is a mere means to the realization of those plan-ends. Synecological system theory is, in the terminology of Dopfer & Potts (2004), a description of the *meso-level* of structured interactions.

An agent-based model of synecological game theory is still in development, but I have developed an outline that describes how such a model could work:

1. Assume a simple number of individuals. Individuals can be initialized with a) a grid topology, b) a social network (scale-free) topology, c) no initial set of connections.
2. Assume a "game-of-interest" played by each individual. Assign randomly. Strategies are based on simple learning algorithms, like testing around some average value.
3. Assume a compatibility structure between certain games via coupling functions with compatible domains and ranges. For abstract games, we can randomize compatibilities. Pull games from a "bag of games" and coupling functions from a "bag of functions" such that synecological games start with games that require no endogenous inputs and end with games that resolve via their coupling functions into real-valued payoffs.

4. INITIALIZATION: (a) assign randomly to agents a “game of interest;” (b) initiate interaction randomly with another player depending on the topology in (1); (c) if a compatible game exists in the partner's repertoire make a connection with the other player and assign a coupling function from the “bag of functions” between the games; (d) repeat steps (b) - (c) for each player until either every other player has been queried, or all games of interest are embedded in a synecological game. (d) There can exist no cycles, that is, game chains must eventually end in real-valued payoffs for all players.
5. EVOLUTION: Play begins. (a) Cycle through starting games first; (b) subsequent games in all synecological chains are played if and only if players have arrived at that point in their synecological chain of gameplay; (c) learning on variable values occurs within the operationalization of each game; (d) at the end of all gameplay, when all resolving games have been played, agents may choose to cut ties with a current player and search for a new compatible player for that stage. New players occasionally enter the system, and with them, new games. This search phase ends when a new compatible player/game has been discovered by all players opting to search.
6. Players who have been “inert” for more than M rounds where M is a parameter may opt to have a new game-of-interest assigned to them.
7. Repeat steps 5-6 for a large number of iterations.

5. Implications for Macroeconomic Modeling: Theoretical Illuminations

In his *Wealth of Nations*, Adam Smith famously explicates the combinatorial beauty of the decentralized production of both needles and wool coats (Smith 1776). By the mid-20th century, macroeconomics had settled around the concept of *technology* being that from which the "extra stuff" of the decentralized combinatorial production process derives: a more efficient machine providing a better ether of production than the same process with a less efficient machine. Empirical macroeconomic modeling exposed the significance of technology to the production process without sufficiently explaining it (Barro 1991).

But by explicitly taking into account the "extra stuff" generated by interaction in an ecology of games framework, we can see the insufficiency of traditional modes of valuing inputs as weighted sums of the value of the final product. I have illuminated that some of the unexplained value of "technology" in the decentralized combinatorial process of production derives from 1) decentralization and 2) combination—that it, the decentralized combinatorial process itself.

Eventually, as macroeconomic models are of the entire system, I envision a synecological systems model that does a few things no standard-use macroeconomic model has done before:

1. **Analytical Polycentricity** - Incorporate the public and political arenas of action in a way that exposes how plans react with and reroute through the public and political arenas both in an exploitative (taking advantage of opportunities as they are created) and an explorative (creating new opportunities for exploitation) fashion.
2. **Endogenous Complexity** - Graduate macroeconomics from studying simple behavior perturbed by random noise to studying complex behavior.
3. **Open-ended Evolution** – Utilize (1) and (2) to model semi-endogenous movement into the adjacent possible economic phase space.

Synecological systems theory intends to answer questions like: why do some policies fail unexpectedly, or take many more resources than expected, or generate the opposite behavior than was intended? Or: how can systems evolve to be robust against predatory behavior, like the collection and use of surveillance capital to alter the choices of individuals to be in line with the interests of system designers? Or: how can it become effectively impossible over time to build affordable housing in growing cities which not only need more affordable housing, but subsidize it? Or: how do robust financial systems become fragile and subject to behavioral cascades when certain channels of synergy are blocked and others enabled to emerge in their absence?

This paper sets out a framework and a mode of analysis that can be used to answer these questions while preserving underlying economic intuition in the form of game theory. Much work lies ahead, namely, getting a proper synecological agent-based model up and running. A basic foundational macroeconomic model requires categorizing the ways in which agent outcomes can be coupled, leaving open the possibility of emergent synergies. Synergies themselves should never be coded into a model, but should emerge or cast shadows on data outputted from simulations, much like the radio signature of as-yet-unknown phenomena in astronomical data.

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