The Industrial Revolution as a Combinatorial Explosion

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ABSTRACT

A remarkably simple combinatorial model of technological change suggests that the Industrial Revolution was a combinatorial explosion in an unchanging process of technological change that began with the origin of our species, or perhaps earlier. In this model, particular causes influenced the timing and location of the Industrial Revolution, but not whether it was going to happen or not. Thus, there is an important sense in which the Industrial Revolution had no special cause.

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I. Introduction

A remarkably simple combinatorial model of technological change suggests that the Industrial Revolution was a combinatorial explosion in an unchanging process of technological change that began with the origin of our species, or perhaps earlier. In this model, particular causes influenced the timing and location of the Industrial Revolution, but not whether it was going to happen or not. Thus, there is an important sense in which the Industrial Revolution had no special cause.

For purposes of this paper, the Industrial Revolution was the spike in global average personal income that kicked in about 1800 C. E. Global average income was stuck below $4.00 a day from the origin of the human species to about 1800 C. E., when it began to rise at super-exponential rates.[1] This pattern of prolonged stasis followed by sudden takeoff is the "hockey stick of economic growth." As our combinatorial model shows, this hockey stick pattern could have been the inevitable product of an unchanging process of cumulative technological evolution that began even before the emergence of anatomically modern humans.

Our explanation builds on Brian Arthur's (2006, 2009) theory of the "combinatorial evolution" of technology. We model combinatorial evolution with a

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[1] The World Bank (1990) declared the international poverty line in 1985 PPP dollars to be $370 per year, or about a dollar a day, which may be roughly translated to about 1.97 2019 PPP dollars per day. (This number is computed by somewhat arbitrarily adjusting for changes in the US PPI as reported by the Bureau of Labor Statistics and picking the range May 1985 to May 2019.) As we note below GDP per capita varied between 450 and 700 1990 Geary-Khamis dollars, which translates to daily values between 1.24 and 1.92 1990 dollars. Those 1990 values roughly translate to 2.17 to 3.37 2019 dollars per day. Thus, the historic values of GDP per capita were above the World Bank's declared poverty line, but still very low indeed by today's standards.

remarkably simple equation for growth in “cambiodiversity.” Following Koppl et al. (2015) we define “cambiodiversity” as the variety of goods traded. If growth in average personal income entails increasing cambiodiversity (Mandeville 1729, Smith 1776, Menger 1871, Young 1928, Hidalgo et al. 2007, Beinhocker 2007, Koppl et al. 2015), then our equation may help to explain why the Industrial Revolution was inevitable once our biological ancestors began making composite tools, perhaps about 300,000 years ago (Ambrose 2001, p. 1751). These hominins bound a sharp stone point to a shaft to produce better tools for hunting and fighting. They were in this way *combining* existing tools to produce new tools. Following Arthur (2006, 2007, 2009) we view cumulative technological change as the production of new goods by modifying existing goods and, importantly, combining and recombining old goods.

Arthur’s model of combinatorial evolution draws on the gradually emerging field of cumulative technological evolution. Adam Smith (1776, I.i.9) explained “improvements in machinery” as resulting in part from “combining together the powers of the most distant and dissimilar objects.” Ogburn’s 1922 book *Social Change With Respect to Culture and Original Nature* is perhaps the first work to significantly develop the idea of technological evolution by combinations of existing tools or goods. Kauffman (1988, 2008, 2016, 2019) discusses cumulative technological evolution though combinations in the more inclusive framework of the evolution of economic webs of new complements and substitutes. Arthur’s contribution develops a comprehensive view of technological evolution via combinations and niche creation in the build out of the economy. Recently, Read and Andersson (2019) raise similar issues in archeology. Fink et al. (2017) and Fink and Reeves (2019), who cite Arthur (2009), have also represented technological change as recombination. Their model has distinct similarities to ours, but assumes that the number of “products” (useful combinations) is fixed *ex ante*. Our more open-ended model seems to give greater scope for the emergence of novelty.

We call our simple combinatorial equation the “TAP equation,” where TAP is an acronym for Theory of the Adjacent Possible. The TAP equation exhibits a long period of relative stasis followed with probability one by a sudden “takeoff” in which cambiodiversity rises super-exponentially (Steel, Hordijk, and Kauffman 2019). There is a small unchanging probability that a randomly chosen good can be modified to produce a new value-enhancing good, a smaller probability that two randomly chosen goods can be combined to produce a new value-enhancing good, an even smaller probability that three randomly chosen goods can be combined to produce a new value-enhancing good, and so on. New useful goods are added to the current stock of existing goods, thereby accumulating ever more technologies that can be created out of what now exists. Thus, over time these small probabilities create an accumulating, hence growing, set of existing goods, causing the number of available goods to grow glacially for a long time then more rapidly.

Due to the glacial growth of technology over thousands of years, wealth stayed very low. Any increase in population level soon drove per capita wealth below the subsistence level and the population died back. Human history was trapped in this Malthusian way for hundreds of thousands of years. Thus, Malthusian population dynamics prevent this slowly accelerating growth from lifting global average income. But with the onset of the Industrial Revolution around 1800 C.E. wealth increased faster than could the population. Thus, the human population could now grow rapidly.

Average income then rises well above earlier levels. The Industrial Revolution was inevitable, though inevitably a long time in coming (Weitzman 1998, Jones 2001, Acemoglu and Zilibotti 1997).

Our explanation of the Industrial Revolution has, of course, limits. Importantly, we do not address the recent demographic shift, whereby increasing wealth induces lower rates of population growth. Nor do we attempt to predict whether technological change will in the future slow down, continue apace, or, perhaps, reach "singularity." Thus, we do not consider whether the "singularity," should it be coming, would be heaven, hell, something else altogether.

Explanations of the Industrial Revolution include, *inter alia*, exploitation of the worker by the capitalist (Marx 1867), Calvinism stimulating the emergence of a unique capitalistic form of economic rationality (Weber 1920, 1927), the emergence of trade-friendly institutions initially brought on by England's Glorious Revolution of 1688 (Dam 2005)[2], the predominance of Christianity in Western culture (Stark 2005), genetically determined increases is parental investment in children. (Golan and Moav 2002), the supposed beneficial eugenic consequences of English primogeniture (Clark 2007), and a shift in the perceived dignity of commercial activity (McCloskey 2010). Each of these explanations appeals to a special cause (or combination of causes) of the Industrial Revolution, and none has emerged as the predominant or consensus view.

We propose a deflationary explanation of the Industrial Revolution. Our explanation *deflates* competing views that depend on some special cause or combination

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[2] North and Weingast (1989) is the usual cite for this claim. They were, however, too circumspect for such an unqualified attribution.

of causes to account for the sudden takeoff of the Industrial Revolution. Takeoff might have occurred in another time or place. Its occurrence in late 18th and early 19th century England is largely accidental and independent of any special cultural, institutional, or genetic causes present uniquely in that time and place. In our model, median income is largely unchanging for a long time before a sudden technological takeoff produces increasingly rapid economic growth. We show that this hockey-stick pattern of economic growth can be explained without appeal to any supposed special causes.

Our explanation of the Industrial Revolution has an obvious affinity to Weitzman (1998) and to unified growth theory in economics (Jones 2001, Acemoglu and Zilibotti 1997). We consider these connections in the discussion section.[3] Notwithstanding important affinities to earlier work, we think our story gives us a new understanding of the Industrial Revolution and its ultimate causes. But we do not believe that it resigns previous work to the dustbin of irrelevance. Rather, we need to rethink earlier explanations in the light of our account. In other words, we should re-think history, not ignore it. Thus, for example, we might keep much of Max Weber's discussion of rationalization while viewing it as more consequence than cause of economic development. The initial spread of Weber's "Protestant ethic" may have owed less to the theological innovations of John Calvin than increases in the opportunity costs of

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[3] We are preparing a separate paper situating our model in modern growth theory. As we suggest below, the basic idea is simply to use the TAP equation to model the level of technology in an otherwise standard unified growth model. In this paper we do precisely that, except that we use a highly simplified growth model in which population is exogenous. Our excuse is that we are here addressing the literature in economic history, and not the distinct if overlapping literature on growth theory.

unproductive activity. This humdrum example illustrates the broader need to rethink prior work in the light of our analysis.

In the classical article “Evolution and Tinkering,” François Jacob said natural selection “works like a tinkerer – a tinkerer who does not know exactly what he is going to produce but uses whatever he finds around him whether it be pieces of string, fragments of wood or old cardboards; in short it works like a tinkerer who uses everything at his disposal to produce some kind of workable object.” Tinkering is jury-rigging. It is, in general, easier to jury-rig some solution to some arbitrary problem with a garage full of “stuff” than a garage with only a small amount of “stuff”. We use this feature of jury rigging in our TAP equation below. More, in general, jury rigging uses the “stuff” in unexpected ways. Unlike the engineer, who needs “tools that exactly fit his project,” the tinkerer “always manages with odds and ends.” Evolution as tinkering has proven a successful metaphor (Jacob 1977, Solé et al. 2003, Henrich 2016, Kauffman 2019). Such tinkering occurs in the biosphere without intelligent search. These are the abundant Darwinian preadaptations, or exaptations, that drive much of evolution and yield ever novel functionalities, (Kauffman 2008, 2019). We apply the idea of tinkering where it is also not metaphoric: the evolution of technology (Arthur 2009). The evolution of the technosphere is driven not by *blind* tinkering, but by intelligent search, by forward-looking, intelligent *human* tinkering. We draw our vision of human tinkering from Arthur (2009), who carefully delineates the delicate balance of intelligent foresight and dumb luck in driving the “combinatorial evolution” of the technosphere.

II. What was the Industrial Revolution?

The term "Industrial Revolution" has been used for a long time by a variety of writers and has, therefore, multiple meanings. In its main uses the term may refer to 1) the new machines, such as the Spinning Jenny, which emerged in England beginning about 1770, or 2) the increased wealth those new machines may have helped to bring about. As we shall see, the social transformations associated with those new machines was also a defining characteristic of the Industrial Revolution for some thinkers, particularly in the earlier literature. More recent controversy has centered on the harm or benefits done to workers and whether the changes in wealth and technology often associated with the new machines was rapid enough to be revolutionary.

It is now well established that real wages for the working classes grew during the Industrial Revolution, though, as Griffin (2018) notes, this increase may not have spread to the country. It is less settled whether such increases improved quality of life during the Industrial Revolution or only with a delay of several decades. Nor is there a firm consensus whether the changes of the Industrial Revolution were rapid enough to be dubbed "revolutionary." It is generally recognized, however, that the Industrial Revolution marked an unprecedented break from the "Malthusian Regime."

Galor and Weil (2000) distinguish a "Malthusian Regime in which technological progress and population growth were glacial by modern standards, and income per capita was roughly constant" (p. 806). Any advance in technology increases output, causing the now enriched population to grow. But this population growth strains the recently increased carrying capacity of the economy, driving average income and population growth rates back down to low levels. This has been the fate of humanity from its origins to the Industrial Revolution, with exceptions being few and local.[4] In the "Post-Malthusian Regime," technological change is rapid enough to outstrip population growth and income per capita grows (p. 807). Finally, in the "Modern Growth Regime" technological change and income growth continue, but now "there is a negative relationship between the level of output and the growth rate of population" (p. 806).

The escape from Malthus was the most important event in human history. And it is this which constitutes, in our view, the essence of the Industrial Revolution. Although early observers noted the increase in wealth and population that coincided with the Industrial Revolution, it was not at first obvious that a change had been made from a Malthusian Regime to one in which average income could rise well above historic levels without collapsing back again. The initial failure to recognize the essential nature of the Industrial Revolution is hardly surprising since the first edition of Malthus famous essay appeared only in 1798. Even in later years, however, debates over the Industrial Revolution tended to focus on the ideological charged issue of its good or evil effects. It may have been inevitable that the importance of the escape from Malthus would be obscured as long as it was contested whether workers benefited from the Industrial Revolution.

The meaning we give the term for our argument, the spike in global average personal income that kicked in about 1800 C. E., seems consistent with the main currents of past usage. In our view, we have noted, the real essence of the Industrial Revolution is the "escape from Malthus." But the history of the term and the debates on worker

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[4] For possible local and partial exceptions see (Broadberyy et al. 2015, Clark 2007), Malanima (2011), van Zanden and van Leeuwen (2012), and Alvarez-Nogal and de la Escosura (2013).

welfare (to which we now turn) might make it inappropriate to *define* the Industrial Revolution as the escape from Malthus.

Bezanson (1922) shows that phrases similar to "industrial revolution" can be found in a French literature tracing back to the earliest years of the nineteenth century. She imputes this early coinage to "a very natural association with the political changes of the French Revolution and the rapid industrial changes" of the period (p. 343). In these French discussions, it seems, the "revolution" could refer to changes in machinery or to changes in social relations brought on by the new machines (Bezanson 1922, p. 347).

The exact phrase "industrial revolution" (in more or less its current meaning) seems to have entered the English language relatively late. Griffin (n.d.) notes, however, earlier authors, including Colquhoun (1814) and Ure (1835), who described the phenomenon in different words.

Engels (1845) uses the exact phrase "industrial revolution" to mean the relatively recent great change in technology. He views the Spinning Jenny, "invented in the year 1764," as the "first invention which gave rise to a radical change in the state of the English workers" (p. 34). Engels makes the same association with the French Revolution that was noted by Bezanson.

Heller (2011) says, "By the 1840s reference to the Industrial Revolution had become part of current English and French usage." But Griffin (n.d.) says, "It was not until the 1840s that the expression began to filter into the English language, and its meaning when it did so was unsettled." Noting Engels' usage she says, "But the influence of Engels on mid-nineteenth-century conceptions of industrialisation was in fact extremely limited." He did, "in time," Griffin, of course, acknowledges, "cast a very long shadow over interpretations of the industrial revolution." But not in the English-language literature of the 1840s. "None of his work was translated from the German until the 1880s, and until that date, was largely passed over by British political economists and social commentators, who remained blissfully unaware of their industrial revolution and newly created industrial proletariat." J. S. Mill used the term "industrial revolution" his *Principles*, first published in 1848. His usage differs, however, from the sense of Engels and Heller and was modified in later editions of the book. This history tends to support Griffin over Heller.

Griffin (n.d.) denies that the very words, "industrial revolution" had their now-common meaning in English until relatively late in the process. "It was not until the end of the nineteenth century, with the work of the social reformer and historian, Arthur [sic] Toynbee, that the term an 'industrial revolution' decisively entered the English language." Because of (Arnold) Toynbee, Griffin reports, the term spread and became a commonplace even with "members of the chattering classes and workers' educational movements."

In his lectures against Henry George, Arnold Toynbee (1884) seems to take it for granted that his audience knows what the "industrial revolution" is. For Toynbee, the Industrial Revolution is evidently something about technology and the factory system. In his more famous Oxford lectures, he says the "Industrial Revolution" was a change in political regime. "The essence of the Industrial Revolution is the substitution of competition for the mediaeval regulations which had previously controlled the production and distribution of wealth" (Toynbee 1892, p. 85). This characterization might seem to suggest that the Industrial Revolution in England could be traced at least as far back as the Glorious Revolution of 1688 and likely before. We are immediately told, however, that Adam Smith's *Wealth of Nations* "appeared on the eve of the Industrial Revolution" (p. 85). By the 1798, when Malthus first published his *Essay on Population*, it was "already in full swing." One's suspicions, then, fall upon the 1780s as starting point of the Industrial Revolution. Toynbee's list of the "chief features" of Industrial Revolution in his Oxford lectures supports his conclusion: There was an Industrial Revolution, and it crushed the poor worker under its mechanized wheels.

It may be that the spread and general acceptance of Toynbee's usage set the stage for subsequent challenges to the very idea of a "revolution" in manufactures. In any event, Clapham (1939) famously argued that industrial change in England prior 1850 had been more evolutionary than revolutionary. Already in 1910 Clapham cast scorn upon the idea of the Industrial Revolution. Rather than a cataclysmic transformation that crushed the worker, there was, in Clapham's view, gradual change in production techniques and (citing Wood 1899) a roughly 42% growth in "industrial wages" from 1790 to 1850 (Clapham 1939, p. 561).

Clapham's "optimism" on wages was a bold stance in its day given that "most of the historians between Marx and Clapham saw the Industrial Revolution as a 'bleak age' for the labouring classes" (Hobsbawm 1963, p. 124). Ashton (1949, p. 19) says, "Most of the economists who lived through the period of rapid economic changes took a somewhat gloomy view of the effect of these changes on the workers." The majority, though not universal, view from the start right down to Clapham was that workers were harmed by the Industrial Revolution.

Clapham disliked the term “Industrial Revolution.” By 1948, however, Ashton (1948, p. 2) could say that it would be “pedantic to offer a substitute.” Ashton shared Clapham’s “optimism” on the effects of the Industrial Revolution on workers’ wealth and welfare, but abandoned any effort to nix the term “Industrial Revolution.”

Nef (1943) attributes the “conventional view of the industrial revolution” to Toynbee’s Oxford lectures (Toynbee 1892). Nef was particularly vexed that Toynbee deceived generations into believing that the Industrial Revolution began in 1760, whereas a more proper dating would be the 1780s (p. 5). As we have seen, however, Toynbee seems to have dated the beginnings of the revolution to the 1780s in perfect agreement with Nef. In fairness to Nef, however, we should note that Toynbee cannot be accused of clarity and consistency on this point.

Nef’s criticism of Toynbee was truly conservative. The “conventional idea of the industrial revolution has interposed itself like a dense fog between us and our traditions,” he grumbles. The “intellectual development which made a revolution possible, if not inevitable,” Nef avers, “can be traced back at least to the Renaissance” (p. 25). And it is “almost inevitable” that the “cost of the industrial revolution” will “outweigh the gain,” Nef warns, “unless mankind can recover what is best in the ancient Christian and humanist traditions” (p. 30).

Toybee’s criticism of the Industrial Revolution (that it produced wealth without wellbeing) and Nef’s criticism of Toynbee illustrate the heavy ideological charge of the topic. This ideological charge helps to explain why workers’ wages have been central to disputes over the Industrial Revolution. While our issue is GDP per capita, there is the related, but distinct, issue of how the relative opulence brought on by the Industrial

Revolution was distributed across the population. The Hartwell-Hobsbawm debate was a central episode in working out the good or bad consequences of the Industrial Revolution for the average person. Hobsbawm was a Marxist and Hartwell a liberal. It is perhaps not surprising, therefore, that they viewed the matter differently. What may be surprising is that something of a consensus emerged from the debate, at least for a time. The Industrial Revolution did lead to improvements in both the workers' wages and their overall living conditions, in this consensus view, but the improvement may not have kicked in until about 1820 or, perhaps, as late as 1845 (Engerman 1994, p. 54).

The Howbsbawm-Hartwell debate culminated in a consensus view that was very different from the damnations of Toynbee and the nostalgic anxiety of Nef. Disagreement remained on an indefinite host of interrelated questions, including whether the worker's improved "standard of living" corresponded to a better "way of life" (Hartwell 1971, p. 57). But by, say, 1970 the predominate view seems to have been that the Industrial Revolution had improved the workers' "standard of living" within at most a hundred years of its onset. In 1994, Stanley Engerman said it was "hard to disagree with the spirit of the Hartwellian conclusion" that the Industrial Revolution improved the lives of most English workers. "Life became longer and in many ways it became better, materially and otherwise" (Engerman 1994, p. 70).

Since the time of Engerman's survey, the debate on standard of living has continued. Clark justly laments, "This debate seems endless" (2005, p.1317, n.5). Recent debate has had, perhaps, a greater tendency to focus on the period before 1850. The question has become how quickly workers partook in the overall increase in wealth. Griffin (2018a, p. 72) says, "Over the past twenty years, economic history has produced a vast literature looking at various elements of living standards, yet we find much the same conclusion repeated over and again: real wages were largely stagnant, while according to all other measures life actually worsened." She characterized the estimates in Feinstein (1998) as "a lodestar for all subsequent scholars seeking to map this terrain" (2018, p. 73).[5] Feinstein's putatively pessimistic view, however, implies that "Average Full-Employment Real Earnings" in Great Britain rose by 35% from 1790 to 1850. And that figure is not so far from the putatively optimistic Wood-Clapham value of 42% for the identical period, which we noted earlier. Feinstein is pessimistic, however, because he thinks this value does not adjust for periods of unemployment. Once that adjustment is made, the overall growth in worker incomes for that period shrinks to a relatively meager 25%. Unfortunately, Feinstein does not fully report his unemployment estimates and describes his estimates as "impressionistic" and "ad hoc." Griffin (2018a, p. 74) describes them as "questionable."

By the end of the twentieth century, then, the pessimistic view had progressed from Toynbee's ardent conviction that the Industrial Revolution had suppressed urban wages to Feinstein's use of "[a]d hoc adjustments" and "impressionistic" methods to support the claim that English wage *increases* were modest in the initial decades of the Industrial Revolution. Recent results such as Clark's important studies (2001, 2005) bolster the optimistic view of worker wages in the Industrial Revolution. Griffin's (2018) puzzlement seems well justified. "Given Clark's more optimistic series and the fact that

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[5] Clark (2005, p. 1318, n. 5) lumps Allen (2001) in with Feinstein as a salient pessimist. We consider Allen's own assessment to be more apposite. "Indeed, the broad perspective of this paper shifts the ground from under both 'optimists' and 'pessimists' in the British standard of living debate. Both positions can find support in the indices reported here, but contrary interpretations are also strengthened in both cases" (Allen 2001, p. 433).

Feinstein's pessimistic conclusions were only weakly supported by his own evidence, it is not self-evident why the picture of stagnant wages before 1850 has achieved almost canonical status within the field" (p. 74).

Ashton's summary statement of 1954 still applies. "Very gradually those who held to these pessimistic views of the effects of industrial change have been forced to yield ground" (Ashton 1954, p. 38). And yet it cannot be said that controversy has ceased. It is endless. In this regard, the situation has changed only a little since Ashton's further remark. "But this does not dispose of the controversy. Real earnings might have risen, it was said, but it was the quality of life and not the quantity of goods consumed that mattered" (Ashton 1954, p. 39). Ashton gives plausible evidence that housing and other living conditions for English workers were improved by industrialization. Tellingly, however, it also included a rather defensive discussion of "responsibility" for the poor quality of worker housing (p. 41 ff.). Ashton's defensive tangent reflects the fact that real wages are easier to assess than quality of life.

Speaking from an avowedly Marxian perspective, Heller (2011) insists that the pessimistic view "has been entirely vindicated by recent research" (p. 198). Heller's lone cite to such "recent research," however, is Szreter and Mooney (1998, p. 104). They do provide evidence that life expectancy fell "in provincial cities" from 35 years in the 1820s to 29 years the 1830s. Even in the pessimistic account of Szreter and Mooney, however, the immiseration of the proletariat was not increasing, but abating after the 1830s. They do not provide estimates for the period before the 1820s. Thus, it seems hard to draw conclusions about the consequences of the Industrial Revolution from their estimates. Their estimated decline in life expectancy applies only to "the growing proportion of the population recruited into the urban industrial workforce" (p. 110) rather than the population as a whole.

Other work seems to support a view less pessimistic than that of Szreter and Mooney. Woods (2000, p. 369), for example, finds a decline in life expectancy of only about a month from the 1820s to the 1830s rather than the six-year decline estimated by Szreter and Mooney. Wrigley and Schofield (1981, p. 230) estimate that life expectancy at birth from 1541 to 1871. Life expectancy rose in England from about 34.2 years in 1761 to 41.3 years in 1871. There was regression from 1831 to1851, when life expectancy sank from about 40.8 to 39.5, but the overall trend was positive. Figure 1 plots their numbers, which were calculated for five-year intervals. It seems fair to say that, some ups and downs notwithstanding, life expectancy had a clear upward trend from its local nadir of 27.9 years in 1731.

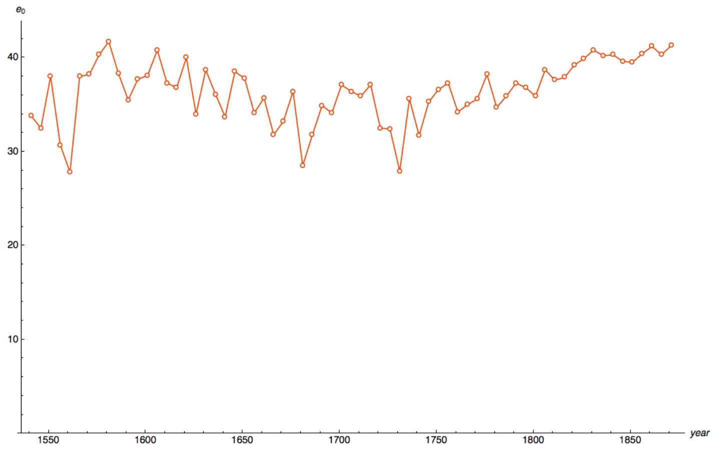

Figure 1

Life expectancy at birth 1541-1871

Data from Wrigley and Schofield 1981, p. 230

Decline in stature has also been a source of pessimistic conclusions. Since the 1970s, stature (i.e. height) has been recognized in economic history as an "index of nutrition" (Fogel et al. 1983) and a measure of the standard of life.[6] (See Lyons et al. 2008 and Williamson and Lyons 2008.) Steckel (2009, pp. 7-10) discusses the determinants of heights and notes the complexity of the relationships between height and other factors such as caloric intake.

Floud et al. (1990) find a generally *upward* trend in heights from 1750 to 1850. This "optimistic" result seems to have been superseded by later work. Komlos (1993 p. 136) found ups and downs along a general *downward* trend in the heights of English men from the 1740s through the 1850s. Floud and Harris (1997 p. 101) say, "The average heights of successive birth cohorts of British men only began to increase consistently from the 1840s onward." Cinnirella (2008) computes a generally negative "secular trend" for average height in Britain from 1740 to 1865 (pp. 338-339). He "finds no support" for the claim by Floud et al. (1990) that "the era of the early industrial revolution led to an improving standard of living" (Floud et al. 1990, p. 151 as quoted in Cinnirella 2008, p. 339).

The literature on stature in England and the UK has supported the view that the Industrial Revolution was bad for the British working class, at least initially. "The stock

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[6] Fogel et al. (1983) is an important early statement. Trussell and Steckel (1978) seems to be the first published result from the group around Fogel. Le Roy Ladurie et al. (1969) spawned a small French literature working on similar lines. The anthropometric literature prompted by Fogel was independent of the earlier French effort.

interpretation is that real wage gains were modest and more than cancelled out by deteriorating urban living conditions" (Griffin 2018, p. 79). Bodenhorn et al. (2017) have noted, however, that the relevant height data is mostly for military recruits and is therefore subject to selection bias. If relatively short men had relatively poor job options, the heights of recruits could have been falling at a time when the true average height was rising. Zimran's (2019) study of US data for "birth cohorts of 1832-1860" seems to confirm the problem of selection bias without overturning the broad conclusion that heights were falling in the US during this time. Overall, the evidence still seems to favor the conclusion that English workers were shrinking 1750-1850. But this conclusion may now seem less definitive, and the amount of shrinkage was probably less than past studies have found.

Shrinkage may also have been, somewhat paradoxically, a product of improved life prospects for English workers and their families. Cinnerella (2008) notes, "It is possible that working-class families during the industrial revolution deliberately chose to have more children at the cost of a lower average nutritional status" (2008, p. 351).[7] In the context of the emergence of agriculture, Locay (1989, pp. 745-747) provides analytical support to Cinnirella's conjecture. Locay explains how technological advance

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[7] This possibility has a certain similarity to Griffin's (2018) suggestion that "men did indeed enjoy higher wages, but this did little to improve the diets of women and children" (p. 79). Like Humphries (1990, 1991, 2013), Horrell and Humphries (1992), and others, these two authors both shift focus to "the family unit rather than the single worker" (Cinnirella 2008, p. 351). Griffin, however, does not adopt a rational-choice perspective and seems to suggest that women were not generally in a position to influence their husband's choices. Of course, different choices will be made within different families, so that any contrasts between the views of Griffin and Cinnirella are a matter of degrees, trends, and averages.

can induce the rational choice of reducing parental food consumption to increase the number of surviving children. Such a choice may well correspond, as Cinnirella notes, to a "lower average nutritional status" per child. A passage in Griffin (2018) suggests that such a deliberate choice may have been made in at least some cases. She surveyed working-class autobiographies from 1750-1850, of which a tiny handful were written by women. "Two writers had believed as children that their mothers stinted their own food so that their children might eat, but two adult female writers indicated that during hard times it was their children rather than themselves who suffered from a want of food" (Griffin 2018, p. 108). Horrell and Oxley (2013) discuss bargaining within the family and note that "remunerated work," though important, is only one factor influencing bargaining strength. Wrigley (1983, p. 144) reports, "Women were marrying much younger at the end of the 'long' eighteenth century than at its beginning and many fewer remained single." Numbers reported in Wrigley et al. (1997, p. 614) reveal that the net reproductive rate (NRR) rose from 1.14 in the period 1711-1756 to 1.39 in the period 1761-1806.[8] However great or small may have been the element of rational choice in family size, the Industrial Revolution seems to have induced an increase in it.

Komlos (1993) also gives the evidence on stature an optimistic spin. We had, he believes, a "demographic expansion" in Europe like other earlier expansions, including a "similar episode of expansion in the sixteenth century" (p. 143). But the greater productivity and wealth of Europe in the 18th century helped to prevent mass starvation. Europe was able to "break through the Malthusian ceiling." Thus, "fewer people fell

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[8] The NRR is the number of daughters a woman may be expected to have in her lifetime, considering the risk that she may die before the end of her child-bearing years.

below the biological minimum than during earlier periods of rapid demographic expansion." It was industrialization, Komlos avers, that made it possible for people to survive "by creating additional income that could be traded for nutrients."

Consistently with Komlos' view, the population of England and Wales expanded greatly in the early decades of the Industrial Revolution. Toynbee notes this increase. "Coming to the facts of the Industrial Revolution, the first thing that strikes us is the far greater rapidity which marks the growth of population" (Toynbee 1892, p. 87). And he quotes Robert Peel (the elder) saying, in 1806, "machinery has given birth to a new population; it as promoted the comforts of population to such a degree that early marriages have been resorted to, and a great increase of numbers has been occasioned by it" (1892, p. 88, n.1).[9] Wrigley's (1969, p. 153) estimates of the population of England and Wales in 1701, 1751, and 1801 imply a growth rate of 0.11% for the first half of the eighteenth century and 0.80% for the second half. In the latter period the population increased almost 50% from 6.140 million to 9.156 million. Estimates of the English population in Wrigley et al. (1997, p. 614) imply annual increases of 0.26% and 0.77% for the same periods. McCloskey (1981) reports that, from 1780 to 1860, "the population increases to an astonishing and unprecedented degree, increasing in England and Wales by about $1\frac{1}{4}$ per cent per year" (p. 105). This increase in human biomass seems to have been enabled by the Industrial Revolution.

We have noted that controversy over optimism and pessimism is endless. Griffin (2018) heaps scorn on the whole question, declaring it "long past its sell-by date" (p. 109). She emphasizes the different effects of the Industrial Revolution on different

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[9] We have not been able to confirm this quote.

populations, noting especially the different experiences of persons in the city and the country and the different experiences of men, women, and children. "The evidence is clear: industrialization ushered in a far more complex, and unequal, society than that which it replaced. It is time to abandon the optimist/pessimist framework and to develop suitably plural, historical approaches and perspectives" (Griffin 2018, p. 110).

Endless debate also continues on whether the Industrial Revolution was revolutionary. Hartwell (1990) notes a "slow rate of growth" literature with important contributors who include "Eric Jones, Rondo Cameron, Nick Crafts, and J. C. D. Clark" (Hartwell 1990, p. 569). For example, J. C. D. Clark (1986, p. 39) insisted, "English society was not *revolutionised*: and it was not revolutionised *by industry.*" Hartwell said in response, "From the very long-term point of view, the revolutionary nature of the changes brought about by industrialization cannot be challenged" (Hartwell 1990, p. 571). Berg and Hudson (1992) vigorously sought to "rehabilitate" the Industrial Revolution. More recently Clark (2007, p. 9) said, "The conventional picture of the Industrial Revolution as a sudden fissure in economic life is not sustainable." Clark's objection is that fluctuations in productivity give us too many candidates for the moment when Britain made a "true break between the Malthusian and modern economies." Plausible candidates include, he tells us, 1600, 1800, "or even" 1860 (2007, p. 9). On the other hand, Clark recognizes and emphasizes the escape from Malthus. That change is the most important thing that has ever happened. And from the extremely long-run point of view we adopt in this paper, it happened very quickly indeed.[10] And yet, if our theory

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[10] Hartwell's "very long-term point of view" spans "several centuries" (Hartwell 1969, p. 14 n. 7). Our "extremely long-run point of view" spans 300,000 years.

is right, one might say that it took two or three hundred millennia to escape from Malthus, which is not so rapid. In any event, the difference between fast and slow is subjective, and thus perhaps not a fit topic for dispute. We have seen that in 1948 Ashton found it "pedantic to offer a substitute" term for "Industrial Revolution." For better or worse, the term is here to stay. And it's meaning, though varying from one writer to another, is connected both to the technological changes that began in the latter half of the eighteenth century and to the increases in average income enabled by those technological changes. Lucase says the Industrial Revolution was the movement from "a traditional world in which incomes of ordinary working people remained low and fairly stable over the centuries" to "a modern world where incomes increase for every new generation." The escape from Malthus forms no part of the definition of the term. But the greater output of the Industrial Revolution could not have enduringly improved the standard of living for most humans had we not escaped from Malthus.

III. The hockey stick of economic growth

After the emergence of anatomically modern humans, perhaps about 200,000 years ago (McBrearty and Brooks 2000, Brown, McDougall, and Fleagle 2012, Stringer 2016, Stringer and Galway-Witham 2017), income levels changed relatively little until the 19th century C.E. when rapid technological change produced a spike in per capita incomes first in Europe and North America and then globally. This spike in incomes supported a corresponding spike in global population. (See Figures Two and Three). This hockey stick of economic growth is the central problem of social science. The central question of social science is, then, Why was there a long period of relative stagnation followed by a sudden takeoff producing rapid technological change, sustained growth, and unprecedentedly high incomes? We show that if technological advance is a result of tinkering and recombination, and the accumulation of successes, then it may proceed slowly for a long time before a combinatorial explosion generates a rapid increase in the variety of goods and a corresponding increase in wealth.

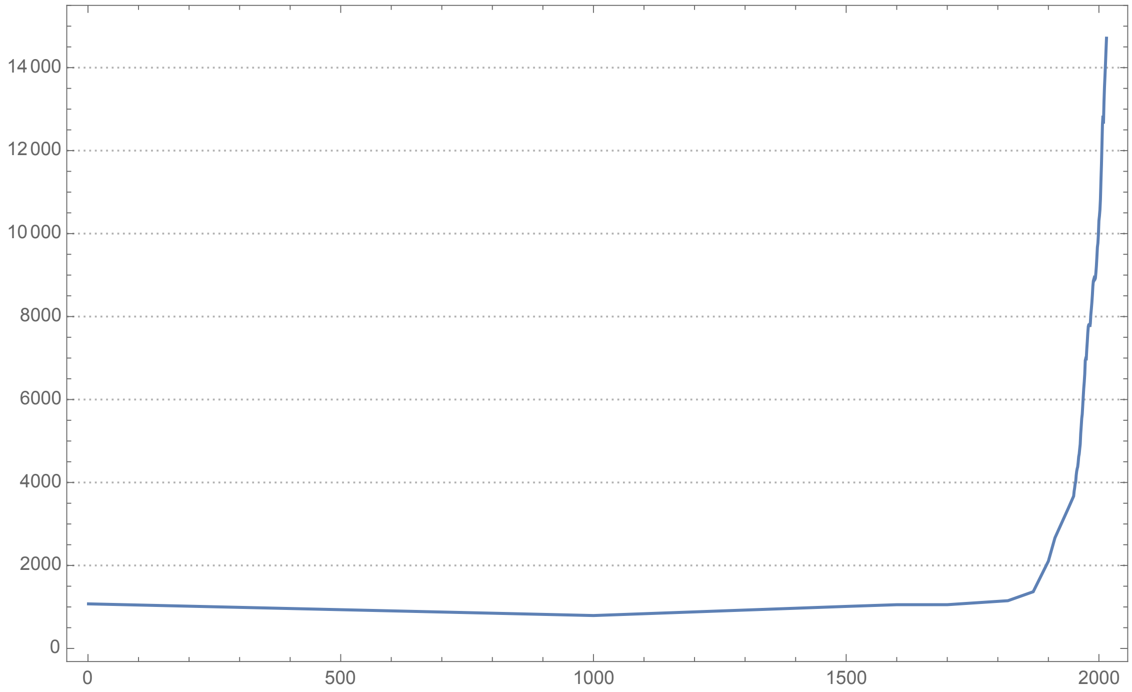

Figure Two

GDP per capita over the long run. Calculated using data from Maddison Project, US Census Bureau, and Kremer (1993).

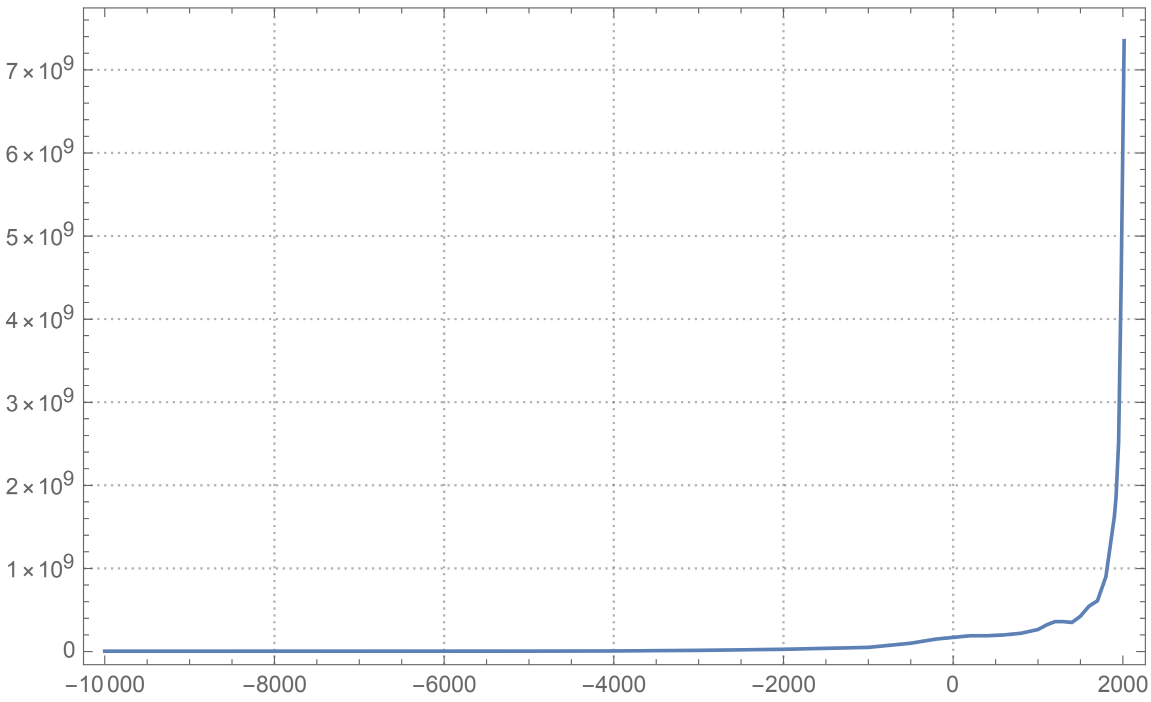

Figure Three

World population from 10,000 BC to the present. Data and estimates derived from US Census Bureau, and Kremer (1993).

Prior to the Industrial Revolution global GDP per capita fluctuated between about 450 and 700 1990 Geary-Khamis dollars. (See Bolt and van Zanden 2014 for a discussion of the Maddison project, which is a vital source of data for this paper.) Already by 1870 (C.E.) this number had risen to about 870, and thus above historical levels. Global GDP per capita in 2008 was over $7,600, which is about 11 to 17 times larger than historical values. In other words, global per capita GDP today is at least an order of magnitude larger than historical levels. In the richer countries, GDP per capita in 2008 varied between about 20,000 and 30,000 1990 Geary-Khamis dollars (Maddison Project). Some evidence suggests that our Pleistocene ancestors may have had a standard of life superior to historical levels prior to the Industrial Revolution (Locay 1989, Hermanussen 2003, Clark 2007). If so, the Industrial Revolution had a far greater effect on incomes than the advent of agriculture and civilization.

Some scholars have found evidence that localized regions such as Northern Italy may have reach relatively high incomes well before the industrial revolution (Malanima 2011, van Zanden and van Leeuwen 2012, Álvarez-Nogal and de la Escosura 2013, Bolt and van Zanden 2014, Broadberry et al. 2015, Dutta et al. 2018). And prior to these relatively recent results, Goldstone (2002) noted several "preindustrial efflorescences" (p. 340) in which the "relatively sharp, often unexpected upturn in significant demographic and economic indices" (p. 333) included "intensive rounds of per capita income growth" (p. 342). All of these episodes of relative wealth, however, were local and not global. Thus, they do not alter the global picture of extended stagnation followed by sudden take off around 1800.

The model

We model technological progress as increasing "cambiodiversity" (Koppl et al. 2015), that is, as increases in the variety of goods. Increasing cambiodiversity is a characteristic feature of economic growth (Mandeville 1729, Smith 1776, Menger 1871, Young 1928, Hidalgo et al. 2007, Beinhocker 2007). In any period, there is a fixed probability that any one good may be modified to produce a new value-enhancing good, and smaller but still fixed probabilities that 2, 3, or *n* goods may be combined to produce a new value-enhancing good. Modifications in Paleolithic hand axes (Ambrose 2001) and in 17th and 18th century American axes (Boyd, Richerson, and Lupp 2013) illustrate how an individual good may be modified to produce a new value-enhancing good. Powered heavier than air flight illustrates how two distinct goods – gliders and internal combustion engines – may be combined to produce a new value-enhancing good, the airplane. In our combinatorial model of technological evolution new types of goods may be generated when tinkerers modify an existing good or cobble together two or more existing goods to come up with some new twist or combination that, with all its imperfections and inelegancies, works well enough to be an improvement.

Let M_t denote the number of distinct types of goods in the economy at time *t*. M_t is the degree of cambiodiversity. Our assumption of fixed probabilities of combining *n* goods to produce a new value-enhancing good leads to the simple combinatorial model given in equation (1) below.

$$M_t = M_{t-1} + P\left(\sum_{i=1}^{M_{t-1}} \alpha_i \binom{M_{t-1}}{i}\right) \qquad (1)$$

where $\binom{M_{t-1}}{i} = \frac{M_{t-1}!}{i!(M_{t-1}-i)!}$, $0 < P\alpha_i < 1$ for $i = 1, 2, \dots, M_{t-1}$, and $\alpha_{i+1} \leq \alpha_i$ for $i = 1, 2, \dots, M_{t-1} - 1$. (In practice, we set $\alpha_i = 0$ for $i > 4$.) $P\alpha_i$ is the probability that if i goods are combined they will result in a new good. We may call P the "master probability of a successful combination," although the probabilities within the model are the $P\alpha_i$ terms. For simplicity, we take equation (1) to describe the net increase in cambiodiversity ($M_t - M_{t-1}$) rather than separately modeling additions and subtractions to the variety of goods under production.

There is a random element in the emergence of new goods, as reflected in the parameters P and α_i. And it may be that many attempts to generate new goods are best viewed as random. But only *value-enhancing* goods will have an enduring place in the econosphere, and it only these value-enhancing goods that we are considering in Equation (1).

The fact that only value-enhancing goods will be produced may not be immediately obvious. But if the purpose of production is consumption, then people will not generally have an incentive to engage in consumption-reducing activities. They will not produce a new and innovative good unless it displaces goods of lower value. Production of the new good will consume resources such as human labor that would otherwise have gone into producing other things including, perhaps, leisure. Thus, whenever a value-enhancing good is added to the system, the overall economic output, GDP, goes up. While errors can and will happen, of course, the tendency is always to produce only such innovative new goods as can cover their opportunity costs with at least some surplus.

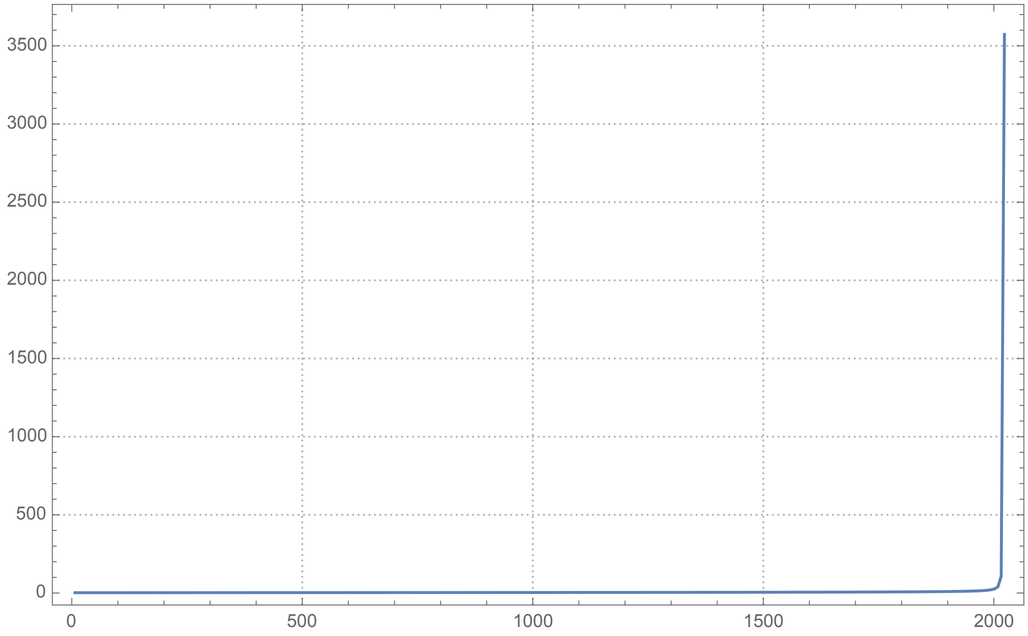

Figure Four

M_t (y-axis), scaled from 0 to 2015 (x-axis).

Our simple model exhibits the characteristic "hockey stick" shape that any explanation of modern economic growth must exhibit. See Figure Four.

Recent analytic results (Steel, Hordijk, and Kauffman 2019) show that the TAP process reaches infinity in finite time with probability one. Thus the TAP process has a singularity. Moreover, the process is super-exponential. Exponential processes do not reach infinity in finite time (Steel, Hordijk, and Kauffman 2019). The TAP process increases glacially for a long time then explodes upward in the hockey-stick we see in Figure Four. The TAP process and equation appears to be new and unique, (Steel, Hordijk, and Kauffman 2019).

We come to our central point. In the TAP cumulative combinatorial process, a glacially long period of largely unchanging values is followed by a sudden takeoff. This behavior emerges from a very simple and unchanging stochastic process. It is not necessary to explain takeoff as the product of some special cause or combination of causes. A low but unchanging value of P will create a long period of stagnation, but sudden takeoff will eventually occur with probability one. It was not the Protestant Ethic, the Glorious Revolution, British primogeniture, or bourgeois dignity that caused the Industrial Revolution. It was slow, grinding probability taking its own sweet time before finally delivering the inevitable combinatorial explosion.

Our model exhibits takeoff in cambiodiversity, which is our measure of technology. Because the rate of increase in cambiodiversity is itself initially increasing, it must produce rapid increases in per capita income, though perhaps with a delay. Similarly, a model with cambiodiversity representing technology exhibits no long-term steady state growth rates of output or capital. Output and capital growth rates are super-exponential instead of constant. Below we add our model of technological change to a simplified version of a standard "unified model" from the economics literature on "modern growth theory." There is diversity in such models and, more generally, there is an indefinite host of particular mechanisms that might translate technological change into increases in population and per capita income. It seems worthwhile, therefore, to first offer some general considerations to explain why technological takeoff will necessarily bring incomes and population up as well.

With an unchanging population, an increase in cambiodiversity would imply a corresponding increase in per capita income. But if the population is initially at low income levels, then the greater abundance brought about by increased cambiodiversity may be translated into an increase in population size without much changing total GDP. The increasing population with roughly constant total GDP drove the population below the subsistence threshold and population die back occurred. And, indeed, before the Industrial Revolution increases in GDP seem to have often, perhaps typically, produced precisely such "Malthusian" income-offsetting increases in population. But technological takeoff produces such a rapid increase in GDP that population cannot rise fast enough to keep up with it. It was thus inevitable that population and average income would both eventually rise precipitously along with cambiodiversity.

Our simple combinatorial model, then, is a sufficient explanation for the Industrial Revolution as defined above. It explains the escape from Malthus. It does not explain the recent demographic transition to lower birth rates occurring after the takeoff in per capita GDP. Prior to the Industrial Revolution, the bulk of the population lived at a relatively low income level, usually dubbed "the subsistence level." In this world, increases in per capita GDP led to increases in population that, in turn, returned per capita GDP to its supposed subsistence level. The rapidly increasing cambiodiversity of the Industrial Revolution caused such rapid increase in GDP that household incomes rose despite population increases. Once incomes rose sufficiently above their subsistence levels, the relationship between income levels and population growth rates changed so that higher incomes now induce falling and not rising population growth rates. This change is the "demographic transition." The rate of world population growth has been declining since about 1970.

Modern growth theory

In the "unified models" of "modern growth theory," the system moves endogenously from a "Malthusian" regime of low growth and steady income to a "Post-Malthusian" regime of higher growth and increasing incomes to, finally, a "Modern Growth regime" of continued technological advance in which, however, population growth no longer increases with income, but instead declines (Galor and Weil 2000, Dutta et al. 2018). In these models, technological change is measured by a scalar whose rate of growth is influenced by the amount of prior knowledge investment. The fundamental form of this process is typically represented as

$$Y_t = A_t K_t^{\beta} L_t^{(1-\beta)} \tag{2}$$

where Y is the output of the economy, K and L are capital and labor. The symbol "A_t" represents "knowledge". Given constant K and L, an increase in A_t increases total productivity of the economy. The theory is "endogenous" growth theory because A_t can change over time endogenously to the model. (Endogenous growth models can be traced to Romer 1990.) In this sort of model, the driver of change (A_t) may be labeled "education," "R&D," "the number of people engaged in producing ideas," (Jones 2001), or something else. In this sort of "idea-based theory of growth," (Jones 2001), resources are diverted from consumption or other productive activities and invested in knowledge production. These models generally assume that "all knowledge resides in the head of some individual person and the knowledge of a firm, or economy, or any group of people is simply the knowledge of the individuals that comprise it" (Lucas 2009). The mechanism linking such investments to technological change is vague or unspecified.

The following model presents a "unified model" of this type. We recognize that our model is as simplistic as it is simple. Our goal is only to illustrate as simply as possible how our model of technological change can be integrated with existing economic models of growth to generate a "unified model" that conforms with the leading facts of economic history.[11]

In our simple discrete time model Y_t is world GDP in period *t*. K_t is the capital stock in period *t*. It is the value of all goods used to generate, ultimately, final output. L_t is the stock of labor. We assume each living person provides the same quantity of labor, which we normalize to one. Thus, L_t is also the population in period *t*. We measure technology by cambiodiversity, M_t. For this simple model, we assume that output is generated by an aggregate production function of the Cobb-Douglas type. Thus,

$$Y_t = M_t K_t^{\beta} L_t^{(1-\beta)} \ , \tag{3}$$

where beta is between 0 and 1.

The capital stock, K_t, is increased by saving, which we assume to be a fixed fraction, *s*, of output. It is diminished by use as, for example, when machines wear out over time. This "depreciation" occurs at the fixed rate delta. Thus, growth in the capital stock is described by the following equation.

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[11] In particular, population is exogenous in this model. As we have indicated earlier, we are working on another paper that integrates the TAP equation with the unified growth literature and uses, therefore, a more satisfactory unified growth model.

$$K_{t+1} = sY_t + (1 - \delta)K_t, \tag{4}$$

where s and δ are between 0 and 1.

In the standard economic models of modern growth theory, the population growth rate is derived from the utility maximizing choices of individuals deciding how may children to have and how much to invest in each child. For the sake of simplicity, and to focus on cambiodiversity, we take population L_t to be exogenous, derived in part from the estimates in (Kremer 1993) and augmented with numbers from the US Census Bureau.

We calibrate the model to reasonably fit growth in total world output from AD 1 to AD 2015, adjusted for inflation and measured in 2011 international dollars (Maddison Project). We require values for 9 parameters to simulate the model. Following the economics literature, we choose 1/3 for capital's share of output (Kremer 1990, Jones and Romer 2010) and values s = 0.25, delta = 0.06.

We assume that α_i is a decreasing function as i increases, and that $\alpha_i = 0$ for $i > 4$. In particular, we assume the decreasing function takes the form

$$\alpha_i = \begin{cases} \frac{1}{(i\,\theta)^{\rho}}, & i \leq 4 \\ 0, & i > 4 \end{cases} \tag{5}$$

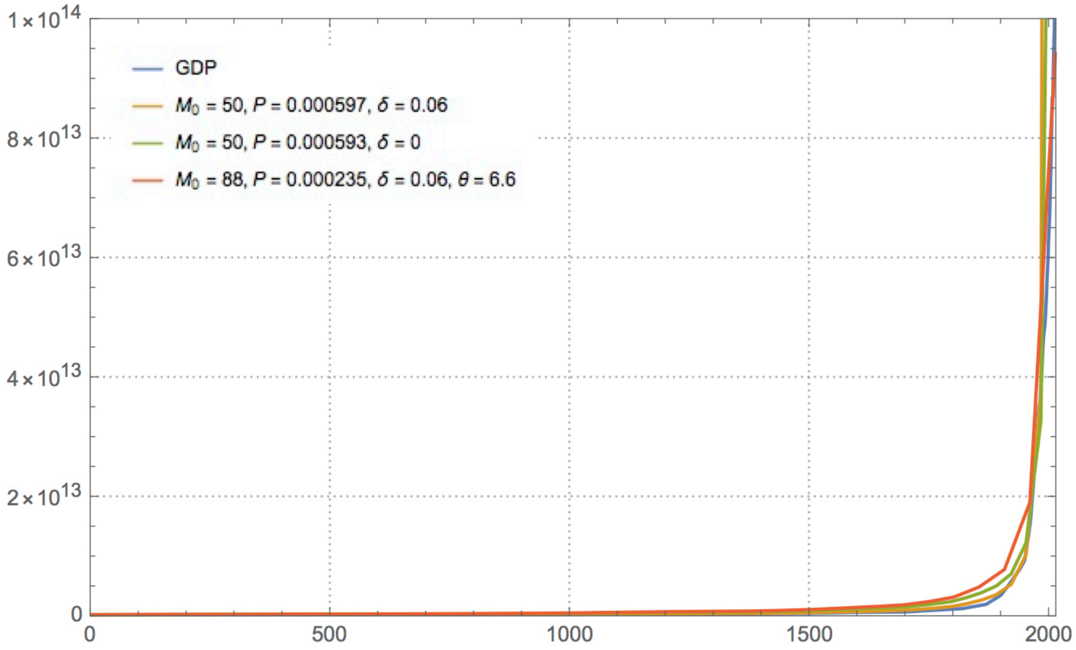
where $\theta > 0, \rho > 0$. The list of parameters is given in Table One.

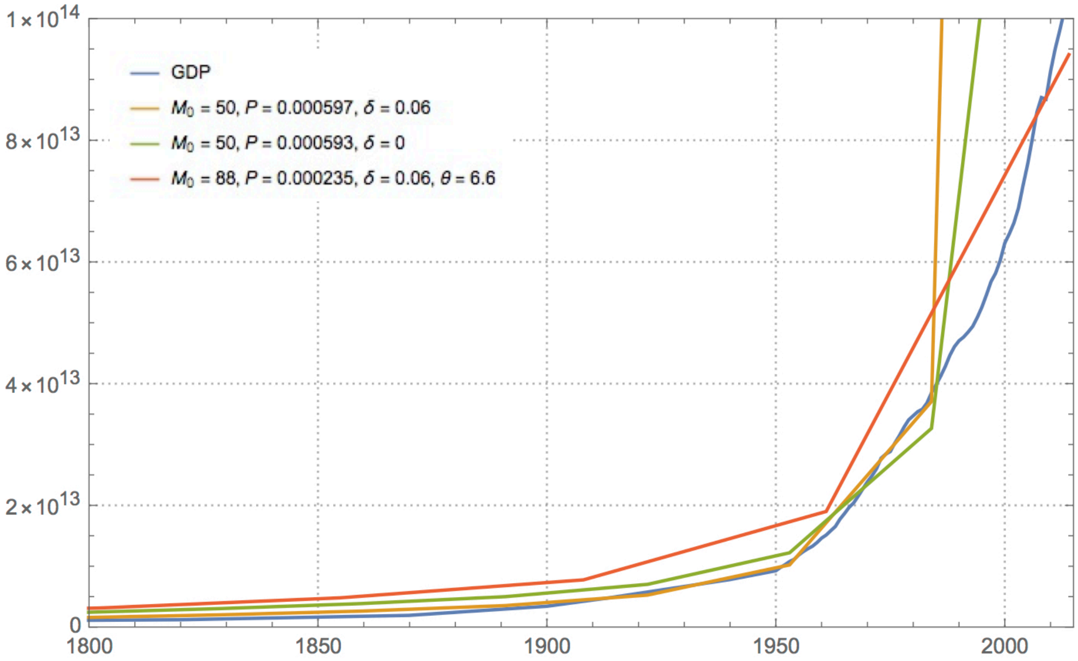
Parameter	Value(s)	Comments
$\boldsymbol{Y_0}$	1.82741×10^{11}	Total world GDP at $t = 0$
$\boldsymbol{M_0}$	50, 88	Number of distinct value-adding goods at $t = 0$
$\boldsymbol{P}$	~0.0006	The master probability of a successful combination
$\boldsymbol{\theta}$	6	$P\alpha_i = P\frac{1}{(i\,\theta)^\rho}$ is the probability that a combination of M_t goods results in a new good
$\boldsymbol{\rho}$	2	$P\alpha_i = P\frac{1}{(i\,\theta)^\rho}$ is the probability that a combination of i goods results in a new good
$\boldsymbol{L_0}$	1.7×10^8	Total world population at $t = 0$
$\boldsymbol{\beta}$	1/3	Capital's share of output
$\boldsymbol{s}$	0.25	Fraction of output re-invested into capital formation
$\boldsymbol{\delta}$	0.06, 0	Capital depreciation rate

Table One

Baseline parameter values of the combinatorial growth model, defined by Equations (3), (4) and (5). Entries with comma-delimited values demonstrate more than one good candidate parameter.

Figure Five shows the estimated progression of total world GDP from AD 1 to the present together with simulated values under three different parameterizations. Note that it is simple to extend the simulation backwards from AD 1, by decreasing the initial number of distinct goods M_0. We consider it important that the model has validity before AD 1. Importantly, the capital stock K_t should not shrink as output Y_t grows. We chose our parameters to ensure the model is coherent prior to AD 1. The value of P in combination with the parameters θ, ρ (which determine α_i) determine how easy or difficult it is to come up with viable products. A higher P, a lower θ, or a lower ρ, all else equal, is correlated with a larger ΔM_t and therefore a larger ΔY_t.


1×10^{14}
8×10^{13}
6×10^{13}
4×10^{13}
2×10^{13}
0
0
500
1000
1500
2000
GDP
$M_0 = 50, P = 0.000597, \delta = 0.06$
$M_0 = 50, P = 0.000593, \delta = 0$
$M_0 = 88, P = 0.000235, \delta = 0.06, \theta = 6.6$


1×10^{14}
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GDP
$M_0 = 50, P = 0.000597, \delta = 0.06$
$M_0 = 50, P = 0.000593, \delta = 0$
$M_0 = 88, P = 0.000235, \delta = 0.06, \theta = 6.6$

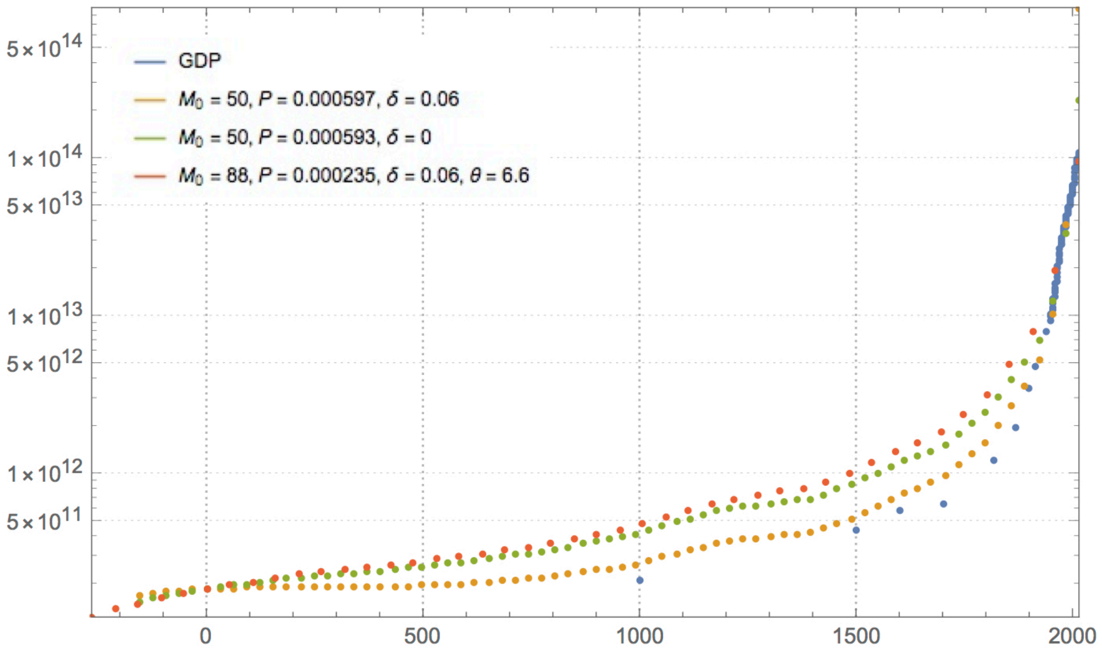

Figure Five

The top graph shows total world GDP (blue) plotted from 1 CE to 2015 CE, together with the parameterization $\{M_0 = 50, \delta = 0.06\}$ (yellow), $\{M_0 = 50, \delta = 0\}$ (green), and $\{M_0 = 88, \delta = 0.06\}$ (red). The middle figure is the same graph, zoomed in to 1800 CE to 2015 CE, to better visually differentiate between the parameterizations. The bottom figure is the same parameterizations plotted on a log GDP scale, from 350 BCE to 2015 CE.

Discussion

We have proposed a deflationary theory of the Industrial Revolution. Our theory deflates rivals that rely on some special cause. In our model, the same stochastic process drives technological change from the earliest days to now. There are other explanations that are at least somewhat deflationary, including at least some contributions to modern growth theory (Galor and Weil 2000, Arifovic, Bullard, and Duffy 1997, Jones 2001, Acemoglu and Zilibotti 1997). These models, however, contain no representation of the central fact of cambiodiversity. They assume relatively modern institutions of market exchange and are therefore not robust to institutions. They assume self-conscious investments in innovation or research rather than tinkering and chance discovery. And they do not have an explicit and satisfying mechanism of technological change. Finally, we know of only one modern growth theory model that has been shown to generate hockey-stick growth in population, GDP, and average income, and this model seems to achieve this result only at the cost of relatively high parameterization (Jones 2001).

The modern growth model closest to our own is probably that of Weitzman (1993), who notes the absence in this literature of a convincing mechanism for the production of new ideas. "Essentially, this approach represents a theory of knowledge production that tries to do an end run around describing the creative act that produces the new ideas." He then develops in some detail a combinatorial approach to the production of new ideas. His analysis has close similarities to our own, although we like to think that our TAP equation may be simpler, more elegant, and more transparent. In any event, he applies combinatorial logic to ideas rather than goods and preserves the idea that knowledge is a product of planned R&D. More importantly, perhaps, he assumes that there must be a maximum rate of knowledge growth. This assumption seems to have led him in a different direction than we have taken. In particular, he focused on the asymptotic growth rate, which is the topic of his "main result," rather than the escape from Malthus.

If our basic model of technological change is correct, then the Industrial Revolution would seem to have been the inevitable consequence of the human propensities to tinker, talk, and trade. The presence of raw-material transfer distances well above likely maximum territorial radius in the Middle Stone Age suggests that grammatical language and long-distance exchange networks emerged at least 130,000 – 140,000 years ago (Marwick 2003, McBrearty and Brooks 2000). Blegen (2017) discusses evidence of long-distance transfer of obsidian occurring about 200,000 years ago or perhaps earlier. He suggests that such transfers could be a result of exchange, of "very mobile hunter-gatherer group[s]," or of both (pp. 14-15). It seems possible that the co-evolution of language and trade was enabled by the arrival of composite tools, which are "conjunctions of at least three techno-units, involving the assembly of a handle or shaft, a stone insert, and binding materials" (Ambrose 2001, p. 1751). Ambrose (2001) dates this arrive to about 300,000 years ago. More recently, however, Wilkins et al. (2012) find evidence of the hafting of stone points, and thus composite tools, about 500,000 years ago. However early long-distance exchange and composite tools may have arrived, we seem to have evidence of an autocatalytic process that began at least 130,000 years ago and in which increases in cambiodiversity enabled the growth of exchange networks, which, in turn, enabled further increases in cambiodiversity and further growth in exchange networks (Smith 1776, Young 1928). Mathusian population dynamics prevented increasing cambiodiversity from inducing increases in personal incomes until the combinatorial explosion of technological change finally overwhelmed population growth, thereby inducing sustained increases in per capita GDP.

Our explanation might seem to neglect the important fact of predation, whereby some persons seize (perhaps violently) goods made by others without offering anything in exchange for them. Such "grabbing," as we may call it, discourages technological change. Grabbing in medieval China, where "property was subject to expropriation by Confucian government officials in the name of the emperor," has been used to explain why inventions did not often become innovations in that country (Lowery and Baumol 2013). A story in Petronius' *Satyricon* vividly illustrates the how grabbing may stifle innovation. An artisan showed Caesar a cup he had made with malleable glass that could not be broken. Once the emperor was satisfied that the artisan had not shared his secret for making malleable glass with anyone else, he had the artisan killed on the spot, "for if the secret were known, we should think no more of gold than of mud" (Lowe 1905). In other words, the innovation threatened to drive down the price of the emperor's gold. Grabbing seems to be as ubiquitous in human life as trade. While we do not separately model grabbing, we do not neglect it either. Grabbing reduces P, and thus the probability of generating a new good. In a more fine-grained analysis we might attempt to plot the ups and down of P as innovation and grabbing interact and exhibit, perhaps, Lotka-Volterra dynamics. In the end, however, the slow and steady power of even a very low P creates takeoff with probability one. (Steel et al. 2019 show that the TAP equation "explodes" with probability one.) Thus, dropping our simplifying assumption of an unchanging P, thereby allowing for differing degrees of grabbing over time, would complicate the analysis without changing the basic contours of our story.

Grabbing is an important aspect of the more general problem of institutions. Economists often attribute different economic outcomes to different social and economic institutions. We neglect institutions. Our timescale is in the scores of millennia. On such a time scale the glacial force is recombination, and institutions fade from view. They are endogenous, and it is tinkering, not the institutional setup, that is the ultimate driver. But on less grand time scales, endogenous institutions are important to explaining outcomes. And on sufficiently short time scales, institutions become exogenous and primary drivers.

As we have seen, the very recent demographic transition produced a slowdown in population growth. This has been a regime change in population growth. It seems worth inquiring whether incomes and cambiodiversity might not be headed toward similar regime changes. Some evidence suggests that we may be approaching a singularity, perhaps around 2050 (Johansen and Sornette 2001). We should also recognize, however, the risk of Chicken Littleism. In a classic article, von Foerster, Mora, and Amiot (1960) estimate that population growth will reach a singularity in 2026. With ironic false precision, they predict "doomsday" on Friday, 13 November 2026. Their humor and irony notwithstanding, they seem to have been sincere in estimating that a population singularity would occur around 2026. As we have seen, however, population growth rates began to fall within about a decade and well before reaching the pitch predicted by their model. It seems, then, both important and difficult to decide whether we will approach a technological singularity, and if so when. Nor is it easy to predict whether the regime change implied by a mathematical singularity would be doomsday or something less dire. In this article, we have adopted the relatively easy task of explaining the past rather than the more daunting task of predicting the future.

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